Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWC 10603
PROGRAMME	:	1 BWC
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Solve the following first order differential equation.

(i) $\dfrac{dy}{dx} = \dfrac{y\cos x}{1+2y^2}$,

(ii) $\dfrac{dy}{dx} = \dfrac{(y\sin x - \sin y)}{(\cos x + x\cos y)}$.

(10 marks)

(b) Solve the given differential equation by using the method of integrating factor.

$$\frac{dy}{dx} - 3y = 6.$$

(4 marks)

(c) A metal bar at a temperature of 100^0 F is placed in a special room at a constant temperature of 0^0 F. After 20 minutes the temperature of the bar is reduced to 50^0 F, inquire

(i) the time it will take the bar to reach a temperature of 25^0 F.
(ii) the temperature of the bar after 10 minutes.

(6 marks)

Q2 (a) Find the general solution for the given second order differential equation.

$$y'' - 36y = 0, \qquad y(0) = 5, \qquad y'(0) = -6.$$

(6 marks)

(b) Find the particular solution for the given second order differential equation by using the method of undetermined coefficient.

$$y'' + 4y' + 8y = \sin x, \qquad y(0) = 1, \qquad y'(0) = 0.$$

(8 marks)

(c) Produce the general solution for the given equation by using the method of variation of parameters.

$$y'' + 16y = 1.$$

(6 marks)

Q3 Given the system of first order differential equations,

$$y_1' = y_1 + 3y_2$$
$$y_2' = 4y_1 + 3y_2$$

(a) Show that the eigenvalues are $\lambda = 5$ and $\lambda = -2$.

(7 marks)

(b) Find the corresponding eigenvectors of A.

(6 marks)

(c) Develop the general solution of the given system.

(7 marks)

Q4 (a) Find $L\{3 + 2t^2 + e^{-2x} \sin 5x\}$.

(6 marks)

(b) Consider the function

$$f(t) = \begin{cases} t-2, & 0 \le t \le 4 \\ 2, & 4 \le t \le 6 \\ 0, & t \ge 6 \end{cases}$$

(i) Sketch the graph of $f(t)$.

(ii) Find the Laplace transform of $f(t)$.

(10 marks)

(c) Analyze the RC circuit problem below and solve it by using Laplace transform.

$$\frac{dI}{dt} + I = 2, \qquad y(0) = 1.$$

(4 marks)

Q5 (a) Find $L^{-1}\left\{\dfrac{1}{(s+1)(s^2+2)}\right\}$.

(8 marks)

(b) By using Laplace transform and then inverse Laplace transform, solve $y' + y = \sin x$ subject to initial condition $y(0) = 1$.

[Hint: Use the result of **Q5**(a) in your solutions]

(12 marks)

- END OF QUESTION -

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Formula

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$ or $a\ddot{y}+b\dot{y}+cy=0$ or $a\dfrac{d^2y}{dx^2}+b\dfrac{dy}{dx}+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay''+by'+cy=f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x)=A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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Formula

The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is $y = y_c + y_p$, and $y_p = uy_1 + vy_2$,

where $u = -\int \frac{y_2 f(x)}{aW}dx$, $v = \int \frac{y_1 f(x)}{aW}dx$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$.

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		