CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	STOCHASTICS PROCESS
COURSE CODE	:	BWA40903
PROGRAMME	:	3 BWA/ 4 BWA
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

CONFIDENTIAL

Q1 (a) For an absorbing Markov chain, the transition probability matrix can be write in canonical form as follow,

$$P = \begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}.$$

Proof that the fundamental matrix can be written as

$$F = (I_m - Q)^{-1}.$$

What is fundamental matrix?

(6 marks)

(b) In a survey investigating changes in housing patterns in a city, it was found that 80% of the population lived in single-family dwellings and 20% in multiple housing of some kind. 10 years later, in a follow up survey, of those who had been living in a single-family dwellings, 70% still did so, but 30% had moved to multiple family dwellings. Of those in multiple family housing, 85% were still living in that type of housing, while 15% had moved to single family dwellings. Assume that these trends continue.

(i) Write a transition matrix of this information.

(ii) What percent of the population can be expected in each category after 20 years?

(iii) What percent of the population can be expected in each category in a long run?

(6 marks)

(c) A Markov chain $X_0, X_1, X_2 \ldots$ has the transition probability matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0.3 & 0.1 & 0.1 & 0.2 & 0.3 \\ 0 & 0 & 1 & 0 & 0 \\ 0.1 & 0.1 & 0.2 & 0.2 & 0.4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Determine the conditional probabilities $\Pr\{X_4 = 3 | X_2 = 1\}$ and $\Pr\{X_{11} = 2 | X_{10} = 1\}$.

(ii) Find the mean time to reach state 4 starting from state 3.

(iii) Starting in state 1, determine the probability that the process is absorbed into state 2.

(9 marks)

(d) A Markov chain $X_0, X_1, X_2 \ldots$ has the transition probability matrix.

$$\begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

Which state(s) is transient? And which state(s) is recurrent? Explain your answer.

(4 marks)

Q2 (a) Given the Q matrix for continuous time Markov chain is

$$\begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix}.$$

(i) Find the eigenvalues and eigenvectors for matrix Q. Hence, calculate $P(t)$.

(ii) From (i), determine the stationary distribution.

(iii) Using Q only, calculate again the stationary distribution.

(9 marks)

(b) Given the Q matrix for continuous time Markov chain is

$$Q = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 2 & -6 & 2 & 2 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{bmatrix},$$

and the product of Q is given as follow.

$$Q^2 = \begin{bmatrix} 13 & -7 & -3 & -3 \\ -14 & 42 & -14 & -14 \\ -3 & -7 & 13 & -3 \\ -3 & -7 & -3 & 13 \end{bmatrix}, \qquad Q^3 = \begin{bmatrix} -59 & 49 & 5 & 5 \\ 98 & -294 & 98 & 98 \\ 5 & 49 & -59 & 5 \\ 5 & 49 & 5 & -59 \end{bmatrix}.$$

The eigenvalues of Q are 0, -4, -4 and -7. Determine $P_{21}(t)$. Hence find $P_{21}(2.5)$.

(8 marks)

(c) For a branching process with offspring distribution given by

$$P_0 = 1/2,\ P_1 = 1/4,\ P_2 = 1/8,\ P_3 = 1/8.$$

(i) Determine the probability that the branching process dies by generation 5 .

(ii) Show that the process ever dies out. Explain your answer.

(ii) If the offspring distribution is $P_0 = 1/4$, $P_1 = 1/4$, $P_2 = 1/8$, $P_3 = 3/8$, find the stationary probability that the branching process will died out.

(8 marks)

Q3 (a)

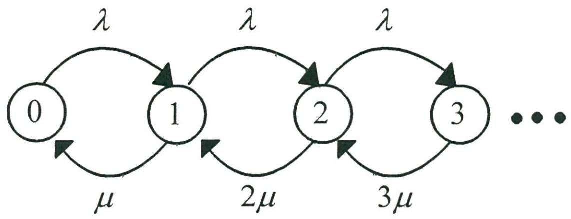

FIGURE Q3(a)

The **FIGURE Q3(a)** shows the state transition diagram for a Markovian queueing model with any queue. By writing the Kolmogorov forward equation, proof that the steady state probability is given by

$$P_n = \frac{\rho^n}{n!}e^{-\rho}, \text{ where } n = 0,1,2,\ldots \text{ and } \rho = \frac{\lambda}{\mu}.$$

(9 marks)

(b) Consider the online ticket reservation system of a company. Assume the customers arrive according to a Poisson process at an average rate of 5 per minute. Also assume that the time taken for each reservation by a computer server follows an exponential distribution with an average rate of 0.2 per hour.

(i) If there are only a computer server, what is the probability that there is no waiting time for a customer who will use the online ticket reservation system of the company?

(ii) Proof that the average number of the customers in the system is $L_s = \frac{\rho}{1-\rho}$.

Hence, find the average time spent in the system, T_s for a customer.

NOTE : You are not allowed to reduce from Q3(c).

(iii) If there are 3 computer servers, what is the probability that there is no waiting time for a customer who will use the online ticket reservation system of the company? In this case, given that $\sum_{n=c}^{\infty} \frac{\rho^n}{c^{n-c}c!} \approx 0.014$.

(11 marks)

(c) For a single server non-Markovian queueing model, the average number of customers in the system in steady state is given by

$$L_s = \rho + \frac{\lambda^2\sigma_S^2 + \rho^2}{2(1-\rho)},$$

where σ_S^2 is the variance of the service time distribution.

(i) Proof that when the service time is exponentially distributed, the average number of the customers in the system is $L_s = \frac{\rho}{1-\rho}$.

(ii) Consider that the customers arrive to a counter according to Poisson process with 4 customers per hour . If the service time for the customer follows Weibull distribution (4, 1/5), where the mean is 0.1813 and variance is 0.002586. Determine L_s and T_q.

(5 marks)

Q4 (a) Suppose that the demand, D, for a product is 30 units per month. The setup cost, K, each time a production run is undertaken to replenish inventory (y) is RM15. The production cost, C, is RM1 per item, and the inventory holding cost, h, is RM0.30 per item per month.

(i) Assuming shortages are not allowed, determine how often to make a production run and what size it should be.

(ii) If shortages are allowed but cost RM3 per item per month, determine how often to make a production run and what size it should be.

(iii) Assuming shortages are not allowed, and the production cost is reduced to RM0.80 per item if the production is more than 100 items. Determine the optimal inventory policy.

(12 marks)

(b) Find the optimal inventory policy for the following three-period model. The demand occurs in discrete units, and starting inventory is $x_1 = 1$. The unit production cost is RM10 for first 2 units and RM20 for each additional unit.

Period, i	Demand D_i (units)	Setup cost K_i (RM)	Holding cost h_i (RM)
1	4	4	2
2	2	6	3
3	3	3	1

(13 marks)

- END OF QUESTION–

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2014/2015 PROGRAMME : 3 BWA, 4BWA
COURSE : STOCHASTIC PROCESS COURSE CODE : BWA40903

Formula

Queueing system	Steady state probabilities
$M/M/1/\infty$	$P_n = (1-\rho)\rho^n$, $n = 0,1,2,\ldots$
$M/M/1/N$	$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}} & if \quad \rho \neq 1 \\ \dfrac{1}{N+1} & if \quad \rho = 1 \end{cases}$, $n = 0,1,2,\ldots N$
$M/M/c/\infty$	$P_n = \begin{cases} \dfrac{\rho^n}{n!}P_0 & 1 \le n \le c \\ \dfrac{\rho^n}{c^{n-c}c!}P_0 & n \ge c \end{cases}$
$M/M/c/c$	$P_n = \dfrac{\rho^n/n!}{\sum_{i=0}^{c} \rho^i/i!}$, $n = 0,1,2,\ldots c$
$M/M/c/K$	$P_n = \begin{cases} \dfrac{\rho^n}{n!}P_0 & 1 \le n \le c \\ \dfrac{\rho^n}{c^{n-c}c!}P_0 & c < n \le K \end{cases}$
$M/M/\infty$	$P_n = \dfrac{\rho^n}{n!}e^{-\rho}$, $n = 0,1,2,\ldots$