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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30602 / BEE 31602
PROGRAMME	:	2 BEJ / 2 BEV / 3 BEJ / 3 BEV / 4 BEJ
EXAMINATION DATE	:	DECEMBER 2014 / JANUARY 2015
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN SECTION A 2. ANSWER **TWO (2)** QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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SECTION A

Q1 Let $y(x,t)$ denotes displacement of a vibrating string. If T is the tension of the string, ω is the weight per unit length and g is acceleration due to gravity, then y satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{Tg}{\omega}\right)\frac{\partial^2 y}{\partial x^2} \quad , \quad 0 \le x \le 2 \; , \; t > 0 .$$

Suppose a particular string with 2 m long is fixed at both ends. By taking $T = 1.5$ N, $\omega = 0.01$ kg/m and $g = 10$ m/s^2, use finite-difference method to solve for y up to second level.

The initial conditions are

$$y(x,0) = \begin{cases} 0.5x & , \quad 0 \le x \le 1 \\ 1 - 0.5x & , \quad 1 \le x \le 2 \end{cases} \qquad \text{and} \qquad \frac{\partial y}{\partial t}(x,0) = x^2 - 2x .$$

Perform all calculations with $h = \Delta x = 0.5$ m and $k = \Delta t = 0.01$ s.

(25 marks)

Q2 The temperature distribution of a heated plate over the square $0 \le x \le 1$, $0 \le y \le 2$ satisfies the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4,$$

with the boundary conditions

$$u(x,0) = x^2 \quad , \quad u(x,2) = (x-2)^2 \quad , \quad 0 \le x \le 1$$

and

$$u(0,y) = y^2 \quad , \quad u(1,y) = (y-1)^2 \quad , \quad 0 \le y \le 2 .$$

Find $u(x, 0.5)$, $u(x, 1)$ and $u(x, 1.5)$ with 2-grid-intervals on the x-coordinate by using finite-difference method.

(25 marks)

SECTION B

Q3 (a) An *RLC* circuit consists of a resistor *R*, an inductor *L* and a capacitor *C* which are connected in series to an alternating voltage source *V*. The current amplitude, i_m is given as follows

$$i_m = \frac{v_m}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}.$$

The parameter values are $R = 140\ \Omega$, $L = 260$ mH, $C = 25\ \mu F$, $v_m = 24$ V and $i_m = 0.15$.

(i) Find the angular frequency, ω by using Newton-raphson method with $\omega_0 = 268$. Do the iteration until it satisfies the convergence criterion, $|f(\omega_i)| < 0.005$.

(8 marks)

(ii) Hence, calculate f, given that ω is related to the frequency f by $\omega = 2\pi f$.

(2 marks)

(b) Consider the following vectors

$$\vec{U} = a\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k},\ \vec{V} = 5\underset{\sim}{i} + b\underset{\sim}{j} + \underset{\sim}{k} \text{ and } \vec{W} = -\underset{\sim}{i} + 2\underset{\sim}{j} + c\underset{\sim}{k}.$$

Given that

$$(\vec{U} \cdot \vec{V}) = 8,\ (\vec{U} \cdot \vec{W}) = 10 \text{ and } (\vec{V} \cdot \vec{W}) = 5.$$

Find the values of a, b and c by using Gauss-seidel iteration method with initial guess, $a_0 = 1.8$, $b_0 = 2.9$ and $c_0 = 3.7$.

(15 marks)

Q4 (a) The velocity, v of air flowing past a flat surface is measured at several distances, y away from the surface (the data is shown in **Table Q4(a)**).

Table Q4(a)

y, m	0	0.002	0.006	0.012	0.018	0.024
v, m/s	0	0.287	0.899	1.915	3.048	4.299

Given shear stress, τ (N/m^2)

$$\tau = \mu \frac{dv}{dy}$$

where a value of dynamic viscosity, μ is assume as 1.8×10^{-5} Ns/m^2. Determine the shear stress at the surface ($y = 0$) by using 2-point forward difference formula. Take $h = 0.002$, 0.012 and 0.024.

(9 marks)

(b) The value of π can be calculated from the following integral

$$\frac{1}{2}\int_{-1}^{1} \frac{4}{1+x^2}\, dx.$$

Approximate π by using trapezoidal rule for $n = 5$.

(7 marks)

(c) An experiment of dropping a ball from the top of a 25-m-tall building is carried out, in order to measure g (the acceleration due to gravity). As the object is falling down, its speed v is measured at various heights, x by sensors that are attached to the building. The data measured in the experiment is given in **Table Q4(c)**.

Table Q4(c)

x, m	5	10	15	20	25
v, m/s	9.85	14.32	17.63	19.34	22.41

The speed of the ball, v as a function of the distance, x is given by $v^2 = 2gx$. Find the value of g at the height of 22 m and 30 m by using Lagrange interpolating polynomial of degree four.

(9 marks)

Q5 (a) Given the following matrix,

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix} \text{ and } v^{(0)} = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^T.$$

Find the largest eigenvalue and its corresponding eigenvector for the matrix A by using power method.

(7 marks)

(b) For a simple RL circuit, real resistors may not always obey Ohm's law. The voltage drop may be nonlinear and the circuit dynamics is described by the following initial-value problem (IVP)

$$L\frac{di}{dt} + R\left[\frac{i}{I} - \left(\frac{i}{I}\right)^3\right] = 0 \quad , \quad i(0) = 0.5,$$

where $i =$ current, $I =$ reference current, $L =$ inductance and $R =$ resistance. The parameter values are given by $L = 1$ H, $R = 1.5$ Ω and $I = 1$ A. Solve the IVP by using fourth-order Runge-Kutta method at $t = 0\,(0.1)\,0.3$ sec.

(9 marks)

(c) The position of a falling object is governed by the following boundary-value problem (BVP)

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} - g = 0 \quad , \quad \text{for } 0 \le t \le 12,$$

where boundary conditions are $x(0) = 0$ and $x(12) = 500$. Given that the parameter values are $c =$ a first-order drag coefficient (12.5 kg/s), $m =$ mass of the falling object (50 kg) and $g =$ gravitational acceleration (9.81 m/s^2). Approximate the position of the falling object, x (m) for $h = 3$ by using finite-difference method.

(9 marks)

- END OF QUESTION -

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FORMULAS

Nonlinear equations

Newton-raphson method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \quad i = 0,1,2,...,n$

System of linear equations

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$

Interpolation

Lagrange polynomial:

$$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0,1,2,\ldots,n \text{ where } L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Numerical differentiation and integration

Differentiation (first derivatives):

2-point forward difference: $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

Integration:

Trapezoidal rule: $\int_a^b f(x)dx \approx \dfrac{h}{2}\left[f_0 + f_n + 2\sum_{i=2}^{n-1} f_i \right]$

Eigenvalue

Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,...$

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Ordinary differential equations

Initial value problems:

Fourth order Runge-Kutta method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$, $k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite-difference method: $y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$, $y_i'' \approx \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$

Partial differential equation

Wave equation: Finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

Poisson's equation: Finite-difference method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \Leftrightarrow \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = f_{i,j}$$