UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	APPLIED STOCHASTICS MODEL
COURSE CODE	:	BWB22303
PROGRAMME CODE	:	BWQ
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) A Markov chain $X_0, X_1, X_2 \ldots$ has the transition probability matrix.

$$\begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.3 & 0.3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.1 & 0.1 & 0.3 & 0.1 & 0.4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Determine the conditional probabilities $\Pr\{X_3 = 2 | X_1 = 3\}$ and $\Pr\{X_8 = 3 | X_6 = 0\}$.

(ii) Find the mean time before absorption starting from state 3.

(iii) Starting in state 0, determine the probability that the process is absorbed into state 2.

(9 marks)

(b) A motor insurance company offers its customers either no discount or 20% discount or 40% discount. A claims free year results in a transition to the next higher discount or in the retention of the maximum discount the following year. On the other hand, claiming in a year results in a reduction to the next lower discount level or the retention of the zero discount in the following year. The probability of no claims in a year can be assumed to be equal to 0.7 in all years. A motorist is offered a 20% discount when he first purchases one such policy (i.e. at 1st January, 2013). Find the probability that at the fourth years (i.e. at 1st January 2017) when the motorist is purchasing the insurance at the same company, the motorist will entitled to a 20% discount.

(6 marks)

(c) What is ergodic markov chain? Write a transition probability matrix that is ergodic but not regular. Determine the following transition matrix is ergodic or not. Explain your answer.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 & 0 \end{bmatrix}$$

(4 marks)

(d) A branching process with offspring distribution given by

$$P_0 = 1/4,\ P_1 = 1/2,\ P_2 = 1/8,\ P_3 = 1/16,\ P_3 = 1/16.$$

(i) Determine the probability that the branching process dies by generation 6 .

(ii) Find the probability that the branching process will died out. If there are 4 independent copies of this branching process at the same time, find the probability that the process ever dies out.

(6 marks)

Q2 (a) Given the Q matrix for continuous time Markov chain is

$$\begin{bmatrix} -5 & 3 & 2 \\ 2 & -4 & 2 \\ 3 & 1 & -4 \end{bmatrix}.$$

Find the eigenvalues for matrix Q. Hence, determine $P_{00}(t)$ and $P_{01}(t)$. Then find $P_{01}(2.5)$.

(12 marks)

(b) Given the Q matrix for continuous time Markov chain is

$$\begin{bmatrix} -4 & 4 \\ 9 & -9 \end{bmatrix}.$$

By writing into system of ordinary differential equations, compute $P(t)$.

(7 marks)

(b) Let say that you wish to open a self-service car wash with some number of stalls. It is estimated that cars will arrive to the car wash at the rate of three per hour. The average time to taken to wash a car is estimated to be 15 minutes. Assume that interarrival times and service time are poisson and exponentially distributed, respectively. Customers in this particular city do not willing to wait. If an arriving customer finds that all stalls are in use, then she will drive away and the business is lost. If all stalls are busy then a customer will balk will probability one. By using the estimates given above, determine the minimum number of stall required so that the long-run probability that a customer will balk is less than 0.5.

(6 marks)

Q3 (a)

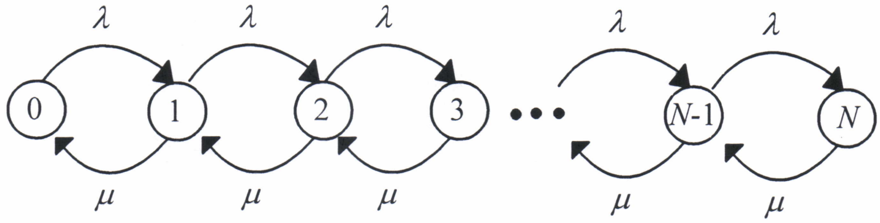

FIGURE Q3(a)M/M/1/N

The **FIGURE Q3(a)** shows the state transition diagram for a single Markovian queueing system. By writing the Kolmogorov forward equation, proof that the steady state probability is given by

$$P_n = \begin{cases} \dfrac{1-\rho}{1-\rho^{N+1}} & if \quad \rho \neq 1 \\ \dfrac{1}{N+1} & if \quad \rho = 1 \end{cases}, \text{ where } n = 0,1,2,\ldots,N \text{ and } \rho = \frac{\lambda}{\mu}.$$

(10 marks)

(b) Consider the online ticket reservation system of a company. Assume the customers arrive according to a Poisson process at an average rate of 0.1 per hour. Also assume that the time taken for each reservation by a computer server follows an exponential distribution with an average rate of 8 per minute.

(i) If there are two computer servers, determine the probability that there is no waiting time for a customer who will use the online ticket reservation system of the company? Given that

$$\sum_{n=c}^{\infty} \frac{(\lambda/\mu)^n}{c^{n-c}c!} \approx \frac{9}{20}$$

(ii) If there are three computer servers, write the expected number that customers need to wait, L_s and the average number of customers in the queue, L_q . You may leave your answer in summation, $\sum$ and P_0.

(9 marks)

(c) For a single server non-Markovian queueing model, the average number of customers in the system in steady state is given by

$$L_s = \rho + \frac{\lambda^2\sigma_S^2 + \rho^2}{2(1-\rho)},$$

where σ_S^2 is the variance of the service time distribution. Consider that the customers arrive to a counter according to Poisson process with 3 customers per minute . If the service time (in second) for the customer follows uniform distribution with parameter 10 to 20. Determine L_s and average waiting time, T_q.

(6 marks)

Q4 (a) Let X_n denotes the number consumable items at the end of a week. In week n, ξ_n items will be consumed with $\Pr\{\xi = 0\} = 0.2$, $\Pr\{\xi = 1\} = 0.5$, $\Pr\{\xi = 2\} = 0.3$. On the weekend, if $X_n > 0$, no buying. If $X_n < 0$, $2 - X_n$ items will be bought. (i.e. $S = 2$, $s = 0$). Set up the corresponding transition probability matrix. How frequently you need to purchase?

(6 marks)

(b) Each day, your opinion on a particular political issue is either positive, neutral or negative. If it is positive today, it is neutral or negative tomorrow with equal probability. If it is neutral or negative, it stay the same with probability 0.5 and otherwise is equally likely to be either of the other two probabilities.

(i) Write the transition probability matrix, P to describe it.

(ii) By finding the stationary distribution, and check for DP (i.e. $(P^{\infty})^T P$). . Determine whether the Markov chain is a reversible Markov chain.

(8 marks)

(c) Given that the transition probability matrix, P.

$$P = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0.5 & 0 & 0.3 \\ 0.1 & 0 & 0.7 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.1 \end{bmatrix}$$

Determine whether the Markov chain is a reversible Markov chain.

(5 marks)

(d) Name two measurements measure by entropy. Given that the transition probability matrix, P.

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Find the entropy rate of stationary markov chain, X_n.

(6 marks)

- **END OF QUESTION–**

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Formula

Queueing system	Steady state probabilities
$M/M/1/\infty$	$P_n = (1-\rho)\rho^n,\ n = 0,1,2,\ldots$
$M/M/1/N$	$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}} & if \quad \rho \neq 1 \\ \dfrac{1}{N+1} & if \quad \rho = 1 \end{cases},\ n = 0,1,2,\ldots N$
$M/M/c/\infty$	$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & 1 \leq n \leq c \\ \dfrac{(\lambda/\mu)^n}{c^{n-c} c!} P_0 & n \geq c \end{cases}$
$M/M/c/c$	$P_n = \dfrac{\rho^n / n!}{\sum_{i=0}^{c} \rho^i / i!},\ n = 0,1,2,\ldots c$
$M/M/c/K$	$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & 1 \leq n \leq c \\ \dfrac{(\lambda/\mu)^n}{c^{n-c} c!} P_0 & c < n \leq K \end{cases}$
$M/M/\infty$	$P_n = \dfrac{\rho^n}{n!} e^{-\rho},\ n = 0,1,2,\ldots$