Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	STATISTICS AND PROBABILITY II
COURSE CODE	:	BWB10303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2016/ JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **FIVE** (5) PAGES

Q1 The Department AW puts projects out on bid and generally estimates what a reasonable bid should be. Call the estimate is b. The Department AW has determined that the density function of the winning (low) bid is,

$$f(y) = \begin{cases} \frac{5}{8}b, & \frac{2}{5}b \le y \le 2b, \\ 0, & \text{elsewhere.} \end{cases}$$

Find $F(y)$ and use it to determine the probability that the winning bid is less the Department AW's preliminary estimate b.

(5 marks)

Q2 For a laboratory assignment, if the equipment is working, the density function of the observed outcomes X is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & otherwise. \end{cases}$$

Find the mean and standard deviation of X.

(6 marks)

Q3 Let Y have a Poisson distribution with mean λ. Find $E[Y(Y\text{-}1)]$ and then use this to show that variance of Y, $\text{Var}(Y) = \lambda$.

(6 marks)

Q4 If Y has a Binomial with n trials and probability of success p, show that the moment-generating function for Y is

$$m(t) = (pe^t + q)^n$$

where $q = 1\text{-}p$.

(5 marks)

Q5 Let consider moment generating function, mgf is given as

$$m(t) = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$$

Find

(a) Mean of *Y*, *E(Y)*

(3 marks)

(b) Variance of *Y*, Var (*Y*)

(5 marks)

Q6 Daily total radiation for a specified location in City A in November has probability density function given by

$$f(y) = \begin{cases} \frac{3}{32}(y-2)(6-y), & 2 \le y \le 6 \\ 0, & \text{elsewhere} \end{cases}$$

with measurement in hundreds of calories. Find the expected daily solar radiation for November.

(5 marks)

Q7 Suppose that $E(\hat{\theta}_1) = E(\hat{\theta}_2) = \theta$, $V(\hat{\theta}_1) = \sigma_1^2$, and $V(\hat{\theta}_2) = \sigma_1^2$. Consider the estimator, $\hat{\theta}_3 = k\hat{\theta}_1 + (1-k)\hat{\theta}_2$. Show that $\hat{\theta}_3$ is an unbiased estimator for θ.

(6 marks)

Q8 A research was conducted to compare the mean number of certain emergency calls per 8-hour shift in two districts of a large city. Sample of 100 8-hour shifts were randomly selected from the records for each of the two regions, and the number of emergency calls was recorded for each shift. The sample statistics are given in **Table Q8**.

Table Q8: Summary statistics for emergency call

	Region	
	A	B
Sample size	100	100
Sample mean	2.4	3.1
Sample variance	1.44	2.64

(a) Estimate the difference in the mean number of emergency calls per 8-hour shift between the two regions in the city.

(2 marks)

(b) Find a bound for error of estimation.

(2 marks)

Q9 If Y has Binomial distribution with parameters n and p, then $\hat{p}_1 = {}^{Y}/_{n}$ is an unbiased estimator of p. Another estimator of p is $\hat{p}_2 = (Y + 1)/(n + 2)$.

(a) Derive the bias of $\hat{p}_2$.

(2 marks)

(b) Derive MSE($\hat{p}_1$) and MSE($\hat{p}_2$).

(4 marks)

(c) For what values of p is MSE($\hat{p}_1$) < MSE($\hat{p}_2$).

(4 marks)

Q10 A state wildlife service wants to estimate the mean number of days that each licensed hunter actually hunts a given season, with a bound on the error of estimation equal to 2 hunting days. If data collected in earlier surveys have shown σ to be approximately equal to 10, how many hunters must be included in the survey?

(5 marks)

Q11 Chronic anterior compartment syndrome is a condition characterized by exercise-induced pain in the lower leg. Swelling and impaired nerve and muscle function also accompany the pain, which is relieved by rest. A group of researchers conducted an experiment involving ten healthy runners and ten healthy cyclists to determine if pressure measurements within the anterior muscle compartment differ between runners and cyclists. The data-compartment pressure, in millimetres of mercury-are summarized in the **Table Q11**.

Table Q11: Data-compartment pressure, in millimetres of mercury

	Runners		Cyclist	
Condition	Mean	Std. dev.	Mean	Std. dev.
Rest	14.5	3.92	11.1	3.98
80% maximal O_2 consumption	12.2	3.49	11.5	4.95

Construct a 95% confidence interval for the difference in mean compartment pressures between runners and cyclists under the resting condition.

(8 marks)

Q12 A precision instrument is guaranteed to read accurately to within 2 units. A sample of four instrument readings on the same object yielded the measurements 353, 351, 351 and 355. Find a 90% confidence interval for the population variance. What assumptions are necessary?

(10 marks)

Q13 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution on the interval $(\theta, \theta+1)$. Let

$$\hat{\theta}_1 = \bar{Y} - \frac{1}{2} \quad \text{and} \quad \hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}$$

(a) Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of θ.

(6 marks)

(b) Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$

(6 marks)

Q14 A survey published in medicine journal reported the number of meters (m) per week swum by two groups of swimmers for those who competed exclusively in breaststroke and those who competed in the individual medley (which includes breaststroke). The number of meters per week practicing the breaststroke was recorded for each swimmer, and the summary statistics are given in **Table Q14**. Is there sufficient evidence to indicate that the average number of meters per week spent practicing breaststroke is greater for exclusive breaststrokers than it is for those swimming individual medley? Test the hypothesis at significance level, $\alpha = 0.01$.

Table Q14: Summary statistics of the number of meters (m) per week swum by two groups of swimmers

	Speciality	
	Exclusively breaststroke	Individual medley
Sample size	130	80
Sample mean	9017	5853
Sample standard deviation	7162	1961
Population mean	μ_1	μ_2

(10 marks)

- **END OF QUESTIONS** -