Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	APPLIED STOCHASTIC MODEL
COURSE CODE	:	BWB22303
PROGRAMME CODE	:	BWQ
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) A Markov chain $X_0, X_1, X_2 \ldots$ has the transition probability matrix.

$$\begin{bmatrix} 0.2 & 0.3 & 0.1 & 0.1 & 0.3 \\ 0 & 1 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Determine the conditional probabilities $\Pr\{X_4 = 4, X_3 = 2 | X_2 = 0\}$ and $\Pr\{X_3 = 3 | X_1 = 2\}$.

(ii) Find the mean time reach state 0 starting from state 2.

(iii) Starting in state 0, determine the probability that the process is absorbed into state 4.

(10 marks)

(b) Bank classifies loans as paid in full (F), in good standing (G), in arrears (A) or as a bad debt (B). Loans move between the categories according to the following transition probability matrix.

$$P = \begin{matrix} F \\ G \\ A \\ B \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 \\ 0.1 & 0.4 & 0.4 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What is the probability that the loans in good standing are eventually become a bad debt? What is the probability that the loans in arrears will eventually paid in full?

(6 marks)

(c) A supermarket offers its customers 5 types of discounts, which is either no discount, 10% discount, 20% discount, 30% discount, or 40% discount for an item every different day. The transition from a type of discount to another type of discount is given by following transition probability matrix.

$$P = \begin{matrix} 0\% \\ 10\% \\ 20\% \\ 30\% \\ 40\% \end{matrix} \begin{bmatrix} 0.3 & 0.1 & 0.1 & 0.1 & 0.4 \\ 0.1 & 0.4 & 0.1 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.5 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 & 0.3 & 0.1 \end{bmatrix}$$

In the long run, find the probability that the item will be sold as no discount.

(4 marks)

(d) For the following transition probability matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 3/4 & 0 & 1/4 & 0 & 0 \\ 0 & 2/5 & 0 & 3/5 & 0 \\ 0 & 0 & 1/4 & 0 & 3/4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(i) Determine the transition matrix is ergodic or not. Explain your answer.
(ii) Determine the transition matrix is regular or not.

(5 marks)

Q2 (a) Given the Q matrix for continuous time Markov chain is

$$\begin{bmatrix} -2 & 1 & 1 \\ 2 & -4 & 2 \\ 1 & 1 & -2 \end{bmatrix}.$$

Find the eigenvalues for matrix Q. Hence, determine $P_{01}(t)$. Then, find $P_{01}(1.5)$ and $P_{01}(\infty)$.

(9 marks)

(b) A small piano store has room to display up to three pianos for sale. Customers come at times of a Poisson process with rate 2 per week to buy a piano, and will buy one if at least one is available. When the store has only one piano left, it places an order for two more pianos, but the order takes an exponentially distributed time with mean 1 week to arrive. While the store is waiting for delivery, sales may reduce to 0 piano. Write down the matrix of transition rates, Q. In the long run, what is the probability that the piano store having 1 piano ?

(6 marks)

(c) Assume that the offspring distribution is Binomial $(4, \frac{2}{3})$.

(i) Find the moment generating function of the offspring distribution, $\phi(s)$. Determine the probability that the branching process dies by generation 4.
(ii) Find the probability that the branching process will died out.

(Hint: For $X \sim B(n, p)$, we have $\Pr(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$)

(6 marks)

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(d) A pure birth process has birth parameters, $\lambda_0 = 1$, $\lambda_1 = 3$. Find the long run stationary distribution.

(4 marks)

Q3 (a)

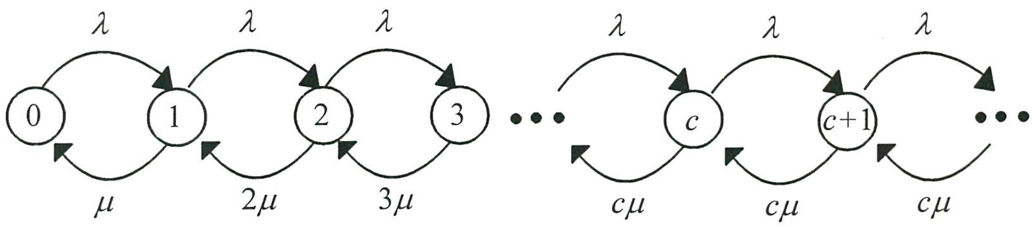

FIGURE Q3(a)M/M/c/∞

The **FIGURE Q3(a)** shows the state transition diagram for a Markovian multiserver queueing system. For $\rho < 1$, by writing the Kolmogorov forward equation, proof that the steady state probability is given by

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & 1 \le n \le c \\ \dfrac{(\lambda/\mu)^n}{c^{n-c} c!} P_0 & n \ge c \end{cases}, \text{ where } \rho = \frac{\lambda}{c\mu}.$$

(8 marks)

(b) Consider the online ticket reservation system of a company, there are infinitely many servers such that every incoming customer finds an idle server immediately. Assume the customers arrive according to a Poisson process at an average of 30 seconds. Also assume that the time taken for each reservation by a computer server follows an exponential distribution with an average of 3 minutes.

(i) Proof that the average number of customers in the system is given by $L_s = \rho$. Hence, find the average time spent in the system.

(ii) During the off peak season, the customers arrive according to a Poisson process at an average of 90 seconds. Hence, the company will provide only C servers where a new user is blocked if all the servers are busy. Enough servers are to be provided to ensure that the probability of the system being full does not exceed 0.0001. How many servers should be provided?

(13 marks)

(c) For a single server M/G/1 queueing system, let X_n be the number of customers at the departure instant of the n-th customer and A_n be a random variable denoting the number of customers who arrive during the service time of n-th customer. Write the transition probability matrix $P_{ij} = P(X_{n+1} = j | X_n = i)$ in term of $a_0, a_1, a_2, \ldots$. For $\rho < 1$, show that the limiting probability is given by

$$v_j = v_0 a_j + \sum_{i=1}^{j+1} v_i a_{j-i+1}, \; j = 0,1,2,\ldots$$

(4 marks)

Q4 (a) Let X_n denotes the number consumable items at the end of a week. In week n, ξ_n items will be consumed with $\Pr\{\xi = 0\} = 0.1$, $\Pr\{\xi = 1\} = 0.3$, $\Pr\{\xi = 2\} = 0.6$. On the weekend, if $X_n > 0$, no buying. If $X_n \le 0$, $2 - X_n$ items will be bought. (i.e. $S = 2$, $s = 0$). Set up the corresponding transition probability matrix. How frequently you need to purchase on the weekend?

(6 marks)

(b) Given that the transition probability matrix, P.

$$P = \begin{bmatrix} 1/3 & 2/3 & 0 \\ 1/5 & 2/5 & 2/5 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

(i) By finding the stationary distribution, and check for DP (i.e. $\left(P^{\infty}\right)^T P$). Determine whether the Markov chain is a reversible Markov chain.

(ii) Check again the reversiblilty of Markov chain by detailed balance.

(8 marks)

(c) Given that the transition probability matrix, P.

$$P = \begin{bmatrix} 0.4 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.1 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.4 \end{bmatrix},$$

Without checking for DP (i.e. $\left(P^{\infty}\right)^T P$), determine whether the Markov chain is a reversible Markov chain. Explain briefly your answer.

(4 marks)

(d) Given that the transition probability matrix, P.

$$P = \begin{bmatrix} 0.4 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.1 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.5 & 0.3 \\ 0.2 & 0.6 & 0 & 0.2 \end{bmatrix}$$

Find the entropy rate of stationary markov chain, X_n.

(7 marks)

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- END OF QUESTIONS -

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Formulae

Queueing system	Steady state probabilities
$M/M/1/\infty$	$P_n = (1-\rho)\rho^n,\ n = 0,1,2,\ldots$
$M/M/1/N$	$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}} & if \quad \rho \neq 1 \\ \dfrac{1}{N+1} & if \quad \rho = 1 \end{cases},\ n = 0,1,2,\ldots N$
$M/M/c/\infty$	$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & 1 \le n \le c \\ \dfrac{(\lambda/\mu)^n}{c^{n-c} c!} P_0 & n \ge c \end{cases}$
$M/M/c/c$	$P_n = \dfrac{\rho^n / n!}{\sum_{i=0}^{c} \rho^i / i!},\ n = 0,1,2,\ldots c$
$M/M/c/K$	$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & 1 \le n \le c \\ \dfrac{(\lambda/\mu)^n}{c^{n-c} c!} P_0 & c < n \le K \end{cases}$
$M/M/\infty$	$P_n = \dfrac{\rho^n}{n!} e^{-\rho},\ n = 0,1,2,\ldots$