SULIT

UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

PEPERIKSAAN AKHIR
SEMESTER II
SESSION 2016/2017

NAMA KURSUS	:	MATEMATIK IV
KOD KURSUS	:	BWM21403
KOD PROGRAM	:	BBV
TARIKH PEPERIKSAAN	:	JUN 2017
JANGKA MASA	:	3 JAM
ARAHAN	:	JAWAB **SEMUA**SOALAN

TERBUKA

KERTAS SOALAN INI MENGANDUNGI **LIMA(5)** MUKA SURAT

SULIT

S1 (a) Selesaikan persamaan pembezaan $x^3 dy - 2y dx = 0$ dengan syarat awal $y(1) = \frac{1}{e}$.

(12 markah)

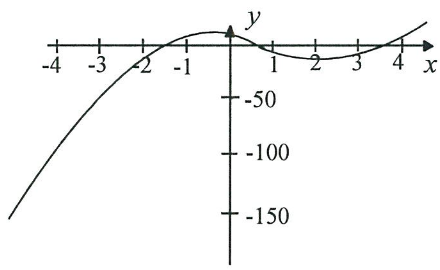
(b) Cari penyelesaian am bagi persamaan pembezaan linear $\frac{dy}{dx} - \frac{4}{x} y = x^4$.

(13 markah)

S2 (a) Cari punca persamaan $x + \ln x = 0$ dengan menggunakan kaedah *Bisection.* Iterasi sehingga $|f(c_i)| < \varepsilon = 0.005$ dengan interval $[0.2, 1]$.

(15 markah)

(b) Diberi graf bagi persamaan $f(x) = 2x^3 - 5x^2 - 7x + 6$ melalui **RAJAH S2**.



Cari punca $f(x)$ paling positif dengan menggunakan kaedah *Newton-Raphson*. Gunakan $x_0 = 3$ dan interasi sehingga $|f(x_i)| < \varepsilon$.

(10 markah)

S3 Diberi sistem persamaan linear seperti di bawah $Ax = b$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 5 \\ -2 & 4 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$$

(a) Tuliskan $A = LU$ dan LU ditakrifkan sebagai kaedah *Doolittle*.

(13 markah)

(b) Melalui $LY = b$, selesaikan Y dengan penggantian hadapan.

(6 markah)

(c) Melalui $Ux = Y$, selesaikan x dengan penggantian ke belakang.

(6 markah)

S4 Selesaikan permsalahan nilai awal order pertama pada $x = 0(0.2)1$ melalui kaedah *Euler*.

$$2\frac{dy}{dx} + 3y = e^{2x}, \text{ dengan syarat awal } y(0) = 1.$$

Diberi penyelesaian yang tepat ialah $y(x) = \frac{1}{7}e^{2x} + \frac{6}{7}e^{-\frac{3x}{2}}$. Cari nilai kesilapan (*error value*).

(25 markah)

- SOALAN TAMAT -

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FORMULAS

Second-order Differential Equation

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Nonlinear Equations

Bisection : $c_i = \dfrac{a_i + b_i}{2}$, $i = 0, 1, 2, \ldots$

Newton-Raphson formula : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0, 1, 2, \ldots$

System of Linear Equations

Crout Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} \begin{pmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{pmatrix}$$

Doolittle Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \begin{pmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{pmatrix}$$

Cholesky Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} \begin{pmatrix} a & b & d \\ 0 & c & e \\ 0 & 0 & f \end{pmatrix}$$

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$, $\forall i = 1, 2, 3, \ldots, n$.

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Interpolation

Lagrange interpolation : $P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$ for $k = 0,1,2,3,\ldots,n$ with $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$

Newton's interpolatory divided-difference formula:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \cdots + f_0^{[n]}(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

Numerical Integration

Trapeziod rule : $\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f_0 + f_n + 2\sum_{i=1}^{n-1} f_i \right)$

Simpson $\frac{1}{3}$ Rule: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f_i \right]$

Ordinary Differential Equation
Initial Value Problem:

Taylor Series Method: $y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!} y''(x_i) + \cdots + \frac{h^n}{n!} y^{(n)}(x_i)$

Classical Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$