Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY I
COURSE CODE	:	BWM 12203
PROGRAMME CODE	:	BNA / BNB / BNC / BND / BNE / BNF / BNG / BNH / BNL / BNM / BNN
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Find the first four nonzero terms of Taylor series for $f(x) = \ln x$, at $x = 1$. (9 marks)

(b) Derive the power series for $\sin x$ if given that

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots.$$

(5 marks)

(c) Calculate $\int_0^1 xe^{4x}\, dx$ using integration by part method. (6 marks)

Q2 (a) Find the following limits. DO NOT use L'Hôpital's rule.

(i) $\lim_{x \to 0} \frac{3x^3 - x}{-x}$. (3 marks)

(ii) $\lim_{x \to 0^-} \frac{2x}{x - |x|}$. (3 marks)

(iii) $\lim_{x \to \infty} x - \sqrt{x^2 - 3x}$. (6 marks)

(b) For the function $f(x)$ graphed in **Figure Q2**, find

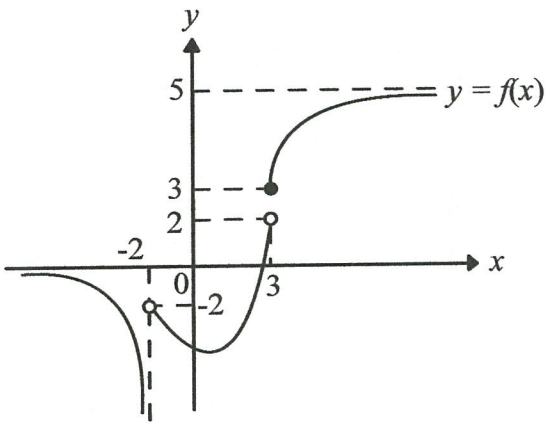

Figure Q2

(i) $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

(1 mark)

(ii) $\lim_{x\to 3} f(x)$.

Is this function continuous at $x = 3$? Give your reason.

(3 marks)

(c) By using the L'Hôpital's rule, find $\lim_{x\to\infty}(2+x)^{\frac{1}{x}}$.

(4 marks)

Q3 (a) Determine the intervals where the graph of $f(x) = x^4 - 4x^3$ is concave up and concave down.

(5 marks)

(b) A plane curve is defined by the parametric equations $x = 2 + 4\cos\theta$ and $y = 3 + 4\sin\theta$, for $0 \le \theta \le 2\pi$. Find

(i) $\frac{dy}{dx}$ when $\theta = \frac{\pi}{4}$.

(6 marks)

(ii) $\frac{d^2y}{dx^2}$.

(4 marks)

(c) If $e^{x+y} = x^3y + \sin(x^2)$, find $\frac{dy}{dx}$ by using implicit differentiation.

(5 marks)

Q4 Find the following integrals.

(a) $\int \frac{x\,dx}{\sqrt{2x-1}}$.

(6 marks)

(b) $\int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x}$.

(6 marks)

(c) $\int \sin^5\left(\frac{x}{3}\right)dx$.

(8 marks)

Q5 (a) A point P is moving along the curve whose equation is $y^2 - x^3 = 17$. When P is at (2, 5), y is increasing at the rate of 2 units/s. How fast is x changing?

(6 marks)

(b) The velocity of a particle, v ms^{-1}, travelling in a straight line, at time t after leaving a fixed point O, is given by $v = 3t^2 - 18t + 32$, where $t \geq 0$. Find the value of t for which the acceleration is zero.

(2 marks)

(c) Solve $\int \frac{x+1}{x^3+6x^2+9x}dx$.

(12 marks)

- END OF QUESTION -

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x\, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\, dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \ |x|<1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \ |x|<1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \ |x|>1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \operatorname{sech}^{-1} |x| + C, \ 0<x<1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \operatorname{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x|<1 \\ \coth^{-1} x + C, & |x|>1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

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Formulae

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$
$\tan x = \frac{2t}{1-t^2}$	$dx = \frac{2dt}{1+t^2}$	$\tan 2x = \frac{2t}{1-t^2}$	$dx = \frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\sin 2x = 2\sin x\cos x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$= 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x\cosh x$
$= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \tan^2 x = \sec^2 x$	$= 2\cosh^2 x - 1$
$1 + \cot^2 x = \csc^2 x$	$= 1 + 2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$	$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$
$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$	$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$
$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$	