UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BWM 12303
PROGRAMME CODE	:	BNA / BNB / BNC / BND / BNE / BNF / BNG / BNH / BNK / BNL / BNM / BNN
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Find a general solution for the first order linear differential equation

$$\frac{dy}{dx}\sin(\pi x) = y\cos(\pi x).$$

(8 marks)

(b) Solve the following initial-value problem (IVP)

$$xy\frac{dy}{dx} = x^2 + y^2, \qquad y(1) = 1.$$

(8 marks)

(c) A cup of coffee is removed from a microwave oven with a temperature of 80°C and allowed to cool in a room with a temperature of 25°C. Five minutes later, the temperature of the coffee is 60°C.

(i) Find the constant k for the cooling process. Given that the Newton's law of cooling is

$$\frac{dT}{dt} = k(T - T_O),$$

where T_O is the temperature of the surrounding medium, k is a constant and t is the time in minutes.

(6 marks)

(ii) Write down the temperature equation of the coffee, for $t \geq 0$.

(1 mark)

(iii) When does the temperature of the coffee reach 50°C?

(2 marks)

Q2 (a) Using variation of parameters method, solve the following differential equation

$$y'' - 6y' + 18y = e^{3x}\operatorname{cosec}(3x).$$

(15 marks)

(b) The equation of motion is governed by

$$\ddot{x} + \frac{a}{M}\dot{x} + \frac{k}{M}x = \frac{F(t)}{M},$$

where k is a spring constant, M is a mass, a is resistance from the surrounding medium and $F(t)$ is an applied force. Suppose that a mass of 2 kg is suspended from a spring with a known spring constant of 8 N/m and allowed to come to rest. It is then set in motion by giving it an initial velocity of 100 cm/sec. Find the position of the mass at any time if there is no external force and no air resistance.

(10 marks)

Q3 (a) Given

$$f(t)=\begin{cases} t^2, & 0\le t<3, \\ 5t, & t\ge 3. \end{cases}$$

Sketch $f(t)$ and describe the function in terms of unit step function.

(5 marks)

(b) The Laplace transform for an unknown function $g(t)$ is given as

$$\mathcal{L}\{g(t)\}=\frac{2}{s^2+s-6}.$$

Find the inverse Laplace transform by using **convolution theorem** in order to determine the function $g(t)$.

(8 marks)

(c) (i) Express

$$\frac{1}{(s-1)(s-2)^2}$$

in partial fractions and show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)^2}\right\}=e^t-e^{2t}+te^{2t}.$$

(7 marks)

(ii) Use the result in (c)(i) to solve the following differential equation

$$y'-y=te^{2t},$$

which satisfies the initial condition of $y(0)=1$.

(5 marks)

Q4 (a) Consider the following initial-value problem (IVP)

$$(1+x^2)\frac{dy}{dx}-xy=0,\quad y(2)=5.$$

Solve the IVP for $2\le x\le 2.2$ and $h=0.1$ by using fourth-order Runge-Kutta method.

(10 marks)

(b) Given the boundary-value problem (BVP)

$$y''+4y=\sin x,\quad 0\le x\le 1,$$

with conditions $y(0)=0$ and $y(1)=0$. Solve the BVP by using finite-difference method with $\Delta x=h=0.25$.

(15 marks)

– END OF QUESTIONS –

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW} dx + A$, $v = \int \frac{y_1 f(x)}{aW} dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Fourth-order Runge-Kutta method

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_i, y_i)$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_i + h, y_i + k_3)$$

Finite difference method

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \quad , \quad y_i'' \approx \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$$