UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	CALCULUS
COURSE CODE	:	BWC 10303
PROGRAMME CODE	:	BWC
EXAMINATION DATE	:	JUNE/JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Let

$$f(t)=\begin{cases}2-2t^2, & t\le -3,\\ 3t-7, & t>-3.\end{cases}$$

Determine,

(i) $\lim\limits_{t\to -3^-} f(t)$.

(ii) $\lim\limits_{t\to -3} f(t)$

(iii) $f(-3)$.

(iv) Does $f(t)$ continuous at $t=-3$? Justify your conclusion.

(5 marks)

(b) Let

$$f(x)=\begin{cases}\dfrac{x^2-16}{x-4}, & x\neq 4,\\[2ex] p, & x=4.\end{cases}$$

Find the value of p so that $f(x)$ continuous for all x.

(5 marks)

(c) Compute the following limits,

(i) $\lim\limits_{x\to 0}\dfrac{x+4}{x^2-16}$.

(ii) $\lim\limits_{x\to\infty}\dfrac{x^3-1}{x^2-1}$.

(iii) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+3}-\sqrt{3}}{x}$.

(10 marks)

Q2 (a) Determine $\dfrac{dy}{dx}$ for the following functions,

(i) $y=\sin\left[\ln\left(\cos x\right)\right]$.

(ii) $x^2y+e^y=x$.

(8 marks)

(b) Evaluate the following limits with the L'Hôpital's rule,

(i) $\lim_{x\to\infty}\frac{\sqrt[4]{x^8-2x^4}}{3x^2+4}$.

(ii) $\lim_{x\to 0}\frac{x^2-3\sin x}{x}$.

(iii) $\lim_{x\to 0}\left((e^x-1)\cdot\frac{1}{\sin x}\right)$.

(12 marks)

Q3 (a) Solve the following integrals,

(i) $\int\frac{(x+5)^2}{5x}\,dx$.

(ii) $\int_0^1 e^{\ln(x+1)^2}\,dx$.

(8 marks)

(b) Use the method of integration by parts to evaluate $\int_{-\pi}^{\pi}\frac{\sin x}{(2x)^{-1}}\,dx$.

(6 marks)

(c) Use the tabular method to evaluate $\int_0^1(x^3+2x)e^{2x}\,dx$.

(6 marks)

Q4 (a) Given $y=x\sqrt{x+1}$,

(i) Show that $\frac{dy}{dx}=\frac{3x+2}{2\sqrt{x+1}}$.

(ii) Hence, evaluate $\int_3^8\frac{3x+2}{\sqrt{x+1}}\,dx$.

TERBUKA

(10 marks)

(b) Let $f(x) = (x^2 - 1)^3$.

(i) Find all critical points of $f(x)$.

(ii) Hence, determine whether the critical points is minimum, maximum or inflection point.

(10 marks)

Q5 (a) Determine the area of the surface that is generated by revolving the line $y = 4x + 2$ between $y = 0$ and $y = \frac{1}{2}$ about y- axis.

(8 marks)

(b) Given $y^2 - 4x^2 = 9$,

(i) Find the curvature κ of the given curve at $x = 2$.

(ii) Find the radius of curvature ρ of the given curve at $x = 1$.

(12 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2017/2018 PROGRAMME CODE :1 BWC

COURSE NAME : CALCULUS COURSE CODE : BWC10303

Formulae

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ \|x\| < 1$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ \|x\| < 1$
$\int \cos x \ dx = \sin x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\int \sin x \ dx = -\cos x + C$	$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$
$\int \sec^2 x \ dx = \tan x + C$	$\int \frac{1}{\|x\|\sqrt{x^2-1}} dx = \sec^{-1} x + C, \ \|x\| > 1$
$\int \csc^2 x \ dx = -\cot x + C$	$\int \frac{-1}{\|x\|\sqrt{x^2-1}} dx = \csc^{-1} x + C, \ \|x\| > 1$
$\int \sec x \tan x \ dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$
$\int \csc x \cot x \ dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ \|x\| > 1$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\|x\|\sqrt{1-x^2}} dx = \text{sech}^{-1} \|x\| + C, \ 0 < x < 1$
$\int \cosh x \ dx = \sinh x + C$	$\int \frac{-1}{\|x\|\sqrt{1+x^2}} dx = \text{csch}^{-1} \|x\| + C, \ x \neq 0$
$\int \sinh x \ dx = \cosh x + C$	$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & \|x\| < 1 \\ \coth^{-1} x + C, & \|x\| > 1 \end{cases}$
$\int \text{sech}^2 x \ dx = \tanh x + C$	
$\int \text{csch}^2 x \ dx = -\coth x + C$	
$\int \text{sech}\, x \tanh x \ dx = -\text{sech}\, x + C$	
$\int \text{csch}\, x \coth x \ dx = -\text{csch}\, x + C$	

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TERBUKA

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2017/2018 PROGRAMME CODE :1 BWC

COURSE NAME : CALCULUS COURSE CODE : BWC10303

Formulae

$\sqrt{k^2 - x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$

$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$

$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2017/2018 PROGRAMME CODE :1 BWC

COURSE NAME : CALCULUS COURSE CODE : BWC10303

Formulae

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi\int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi\int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$

TERRUKA