UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	OPTIMIZATION TECHNIQUES I
COURSE CODE	:	BWA 40603
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JUNE / JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Let the function $f:\Re^n \to \Re$ be a real-valued function that is to be minimized. Suppose that the function f is twice continuously differentiable. Therefore, a quadratic function can be fit through $x^{(k)}$ to match the first and second derivatives of the function f.

Write this quadratic function, say $q(x)$, and show that the minimizer of this quadratic function, which satisfies the first-order necessary condition, can give the following equation:

$$x^{(k+1)} = x^{(k)} - (\nabla^2 f(x^{(k)}))^{-1} \nabla f(x^{(k)}).$$

(8 marks)

(b) Consider the following function:

$$f(x_1, x_2) = 2x_1x_2 + 2x_1 - x_1^2 - 2x_2^2.$$

Apply the Newton's method to find the minimizer of f given that the initial value is $(x_1^{(0)}, x_2^{(0)}) = (-1, 1)^T$.

(12 marks)

Q2 (a) Prove the following proposition:

Proposition:
If $\{x^{(k)}\}_{k=0}^{\infty}$ is a steepest descent sequence for a given function $f:\Re^n \to \Re$ and if $\nabla f(x^{(k)}) \neq 0$, then $f(x^{(k+1)}) < f(x^{(k)})$.

(6 marks)

(b) Use the method of steepest descent to find the minimizer of

$$f(x_1, x_2, x_3) = (x_1 - 4)^4 + (x_2 - 3)^2 + 4(x_3 + 5)^4.$$

Given that the initial point is $x^{(0)} = [4, 2, -1]^T$. Perform three iterations and provide your answer in three decimal places.

(14 marks)

Q3 (a) Suppose the original range $[a_0, b_0]$ is of unit length as shown in **Figure Q2 (a)**. Then, to have only one new evaluation of f it is enough to choose ρ so that

$$\rho(b_1 - a_0) = b_1 - b_2 .$$

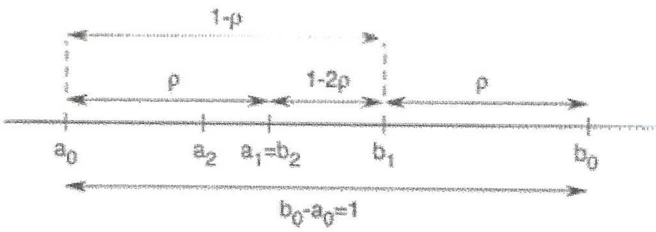

Figure Q2 (a): Unit Length

(i) Write the corresponding quadratic equation, given that $b_1 - a_0 = 1 - \rho$ and $b_1 \quad b_2 - 1 \quad 2\rho$.

(3 marks)

(ii) Obtain the solution for this quadratic equation, given $\rho < \frac{1}{2}$.

(5 marks)

(b) Consider the following function

$$f(x) = x^4 - 14x^3 + 60x^2 - 70x .$$

Use the golden section search method to find the value of x that minimizes the function $f(x)$ in the interval [0, 2]. Perform the calculation until this value of x within a range of 0.3.

(12 marks)

Q4 (a) Consider a quadratic function given by

$$f(x)=\frac{1}{2}x^{\mathrm{T}}Qx-b^{\mathrm{T}}x$$

where $x\in\Re^{n}$, $b\in\Re^{n}$, and $Q\subset\Re^{n\times n}$ is a symmetric positive definite matrix.

(i) Determine the gradient of f.

(2 marks)

(ii) Obtain the unique minimizer of f.

(3 marks)

(b) Consider the scalar function for the quadratic function $f(x)$ is defined as follows:

$$\phi_k(\alpha)=f(x^{(k)}-\alpha\cdot g^{(k)}),\ \text{where}\ g^{(k)}=\nabla f(x^{(k)}).$$

(i) Show that the gradient of ϕ_k is given by

$$\phi_k'(\alpha)=(x^{(k)}-\alpha g^{(k)})^{T}Q(-g^{(k)})-b^{T}(-g^{(k)}).$$

(5 marks)

(ii) Prove that the minimizer $\alpha_k\geq 0$ for the scalar function $\phi_k(\alpha)$ is given by

$$\alpha_k=\frac{g^{(k)\mathrm{T}}g^{(k)}}{g^{(k)\mathrm{T}}Qg^{(k)}}.$$

TERBUKA (10 marks)

Q5 (a) Suppose that the Hessian matrix $F(x)$ of the objective function f is constant and independent of x. In other words, the objective function is quadratic with Hessian $F(x) = Q$ for all x, where $Q = Q^{\mathrm{T}}$.

Express that $\Delta g^{(k)} = Q\Delta x^{(k)}$, where g is the gradient of the objective function f.

(7 marks)

(b) Prove the following theorem.

Theorem:
Consider a quasi-Newton algorithm applied to a quadratic function with Hessian $Q = Q^{\mathrm{T}}$ such that for $0 \le k < n-1$,

$$H_{k+1}\Delta g^{(i)} = \Delta x^{(i)},\ \ 0 \le i \le k,$$

where $H_{k+1} = H_{k+1}^{\mathrm{T}}$. If $\alpha_i \ne 0,\ 0 \le i \le k,$ then $d^{(0)}, \cdots, d^{(k+1)}$ are Q-conjugate.

(13 marks)

TERBUKA

– END OF QUESTIONS –