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# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2017/2018

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|---|---|---|
| COURSE NAME | : | ORDINARY DIFFERENTIAL EQUATIONS |
| COURSE CODE | : | BWC 10603 |
| PROGRAMME CODE | : | BWC |
| EXAMINATION DATE | : | JUNE/JULY 2018 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS |

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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**Q1** (a) Solve the following differential equations and initial value problem.

(i) $\dfrac{dy}{dx} = \dfrac{5x+4y}{8y^3-4x}.$

(ii) $\dfrac{dy}{dx} + 2(x+1)y^2 = 0$ with $y(0) = -\dfrac{1}{8}.$

(12 marks)

(b) A cake is removed from an oven at 100°C and left to cool at room temperature, which is 24°C. After 30 minutes the temperature of the cake is 60°C. Determine the time taken to reach 40°C?

(8 marks)

**Q2** (a) Find the general solution of the following differential equations.

(i) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 6 + 5\sin x.$

(ii) $\dfrac{d^2y}{dx^2} - 10\dfrac{dy}{dx} + 25y = 6e^{5x}.$

(12 marks)

(b) A body with mass 250 g is attached to the end of a spring that is stretched 25 cm by a force of 9 N. At time t = 0 the body is pulled 1 m to the right, stretching the spring, and set in motion with an initial velocity of 5 m/s to the left. Obtain

(i) $x(t)$ in the form of $C\cos(\omega_0 t - \alpha)$.

(ii) the amplitude and period of motion of the body.

(8 marks)

**Q3** (a) Find a power series expansion for the solution of the initial value problem

$$\frac{dy}{dx} = x + y \quad \text{with} \quad y(0) = 1.$$



(12 marks)

(b) A series solution of a certain differential equation is given by

$$y(x) = c_0 \sum_{n=0}^{\infty} \frac{n+1}{3^n} x^n .$$

Find its radius of convergence. What does it mean by a statement "The series is converged "?

(8 marks)

**Q4** For **Q4** please refer to **Table Q4.**

(a) Calculate the Laplace transform of the function below by using the definition of the Laplace transform.

$$f(t) = t^2$$

(8 marks)

(b) Use Laplace transform to solve the initial value problem

$$x'' - 6x' + 8x = 2 \quad \text{with} \quad x(0) = x'(0) = 0.$$

(12 marks)

**Q5** Apply the eigenvalue method to obtain solutions for a given system of first order differential equations with initial values below.

$$y_1' = 2y_1 - 5y_2.$$
$$y_2' = 4y_1 - 2y_2.$$
$$y_1'(0) = 2.$$
$$y_2'(0) = 3.$$

(20 marks)

**– END OF QUESTIONS –**

## FINAL EXAMINATION

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### Table Q4: Laplace Transform

Definition $\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$

| $f(t)$ | $F(s)$ | $f(t)$ | $F(s)$ |
|---|---|---|---|
| $a$ | $\frac{a}{s}$ | $H(t-a)$ | $\frac{e^{-as}}{s}$ |
| $e^{at}$ | $\frac{1}{s-a}$ | $f(t-a)H(t-a)$ | $e^{-as}F(s)$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ | $\delta(t-a)$ | $e^{-as}$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ | $f(t)\delta(t-a)$ | $e^{-as}f(a)$ |
| $\sinh at$ | $\frac{a}{s^2-a^2}$ | $\int_0^t f(u)g(t-u)\,du$ | $F(s)\cdot G(s)$ |
| $\cosh at$ | $\frac{s}{s^2-a^2}$ | $y(t)$ | $Y(s)$ |
| $t^n,\ n=1,2,3,\ldots$ | $\frac{n!}{s^{n+1}}$ | $y'(t)$ | $sY(s)-y(0)$ |
| $e^{at}f(t)$ | $F(s-a)$ | $y''(t)$ | $s^2Y(s)-sy(0)-y'(0)$ |
| $t^n f(t)$, $n=1,2,3,\ldots$ | $(-1)^n \frac{d^n}{ds^n}F(s)$ | | |

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