UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	CALCULUS
COURSE CODE	:	BWC 10303
PROGRAMME	:	BWC
EXAMINATION DATE	:	DECEMBER 2018/JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) **Figure Q1(a)** shows a function of $f(x) = \frac{1}{x}$.

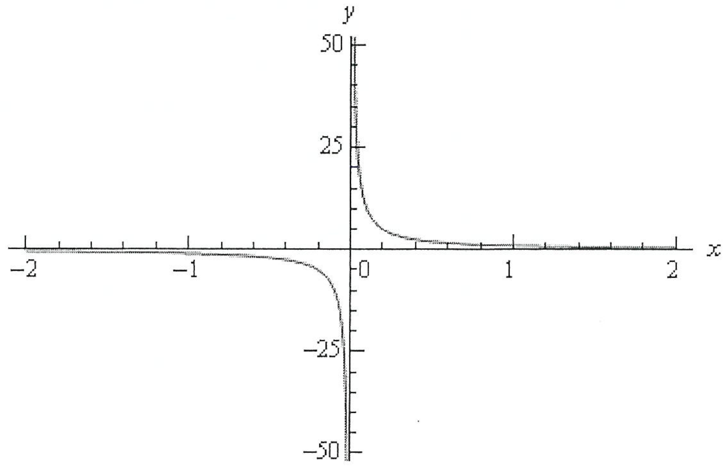

Figure Q1(a)

(i) Find $\lim_{x \to 0^+} f(x)$.

(1 mark)

(ii) Find $\lim_{x \to 0^-} f(x)$

(1 mark)

(ii) Find $f(0)$.

(1 mark)

(b) Referring to **Figure Q1(a)**, is the function continuous at $x = 0$? Justify your answer.

(2 marks)

(b) Differentiate the following functions with respect to x.

(i) $y = \dfrac{\cos x}{1 + \sin x}$.

(2 marks)

(ii) $y = x^{\ln x}$.

(5 marks)

(c) If $(\cos x)y^2 - 2x^2 = e^{xy}$, find $\dfrac{dy}{dx}$ in terms of x and y.

(8 marks)

Q2 (a) Let $f(x) = (x^2 - 1)^3$.

(i) Find all critical points of $f(x)$.

(5 marks)

(ii) Hence, determine whether the critical points is minimum, maximum or inflection point.

(5 marks)

(b) Using L'Hopital's rule, find the limit of the followings.

(i) $\lim_{x \to \infty} (e^x + 1)^{-2/3}$

(5 marks)

(ii) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

(5 marks)

Q3 (a) Evaluate these integrals.

(i) $\int \frac{dx}{x^{-6}}$.

(3 marks)

(ii) $\int \frac{e^{5x}}{e^{3x}} dx$.

(3 marks)

(iii) $\int_0^{\pi} \frac{\sin 2x}{2\cos x} dx$.

(4 marks)

(b) Given $y = x\sqrt{x+1}$.

(i) Show that $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{x+1}}$.

(5 marks)

(ii) Hence, evaluate $\int_3^8 \frac{3x+2}{\sqrt{x+1}} dx$.

(5 marks)

Q4 (a) By using proper substitution, evaluate $\int \frac{x^2}{\sqrt[4]{x^3+2}} dx$.

(5 marks)

(b) Find $\int x^2 e^{-x} dx$.

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(5 marks)

(c) Show that $\int \frac{3x+5}{(x+1)(x-1)^2}\,dx = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + \frac{4}{x-1} + c$.

(10 marks)

Q5 (a) Find $f'(x)$ for $f(x) = \sin^{-1}(\ln x)$.

(5 marks)

(b) Find area of the surface that is generated by revolving the curve $y = \sqrt[3]{3x}$ between $y = -1$ and $y = 0$ about the y-axis.

(10 marks)

(c) Find the curvature if $x = \cos t$ and $y = \ln 2t$ at $t = \pi$.

(5 marks)

- END OF QUESTIONS –

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FORMULAE

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ \lvert x \rvert < 1$
$\int \frac{1}{x} dx = \ln\lvert x \rvert + C$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ \lvert x \rvert < 1$
$\int \cos x \ dx = \sin x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\int \sin x \ dx = -\cos x + C$	$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$
$\int \sec^2 x \ dx = \tan x + C$	$\int \frac{1}{\lvert x \rvert \sqrt{x^2-1}} dx = \sec^{-1} x + C, \ \lvert x \rvert > 1$
$\int \csc^2 x \ dx = -\cot x + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{x^2-1}} dx = \csc^{-1} x + C, \ \lvert x \rvert > 1$
$\int \sec x \tan x \ dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$
$\int \csc x \cot x \ dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ \lvert x \rvert > 1$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{1-x^2}} dx = \operatorname{sech}^{-1} \lvert x \rvert + C, \ 0 < x < 1$
$\int \cosh x \ dx = \sinh x + C$	$\int \frac{-1}{\lvert x \rvert \sqrt{1+x^2}} dx = \operatorname{csch}^{-1} \lvert x \rvert + C, \ x \neq 0$
$\int \sinh x \ dx = \cosh x + C$	$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & \lvert x \rvert < 1 \\ \coth^{-1} x + C, & \lvert x \rvert > 1 \end{cases}$
$\int \operatorname{sech}^2 x \ dx = \tanh x + C$	
$\int \operatorname{csch}^2 x \ dx = -\coth x + C$	
$\int \operatorname{sech} x \tanh x \ dx = -\operatorname{sech} x + C$	
$\int \operatorname{csch} x \coth x \ dx = -\operatorname{csch} x + C$	

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k \tan\theta$	$x = k \sinh\theta$
$\sqrt{x^2-k^2}$	$x = k \sec\theta$	$x = k \cosh\theta$
$\sqrt{k^2-x^2}$	$x = k \sin\theta$	$x = k \tanh\theta$

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TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2} x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$2 \sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$$

$$2 \sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$$

$$2 \cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 2\cosh^2 x - 1$$

$$= 1 + 2\sinh^2 x$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

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FORMULAE

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S = 2\pi\int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S = 2\pi\int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$