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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	CALCULUS I
COURSE CODE	:	BWA 10203
PROGRAMME CODE	:	BWA / BWQ
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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Q1

(a) Sketch the graph of $f(x)=\begin{cases} 2, & x \le -2 \\ -3x, & -2 < x < 0 \\ 2, & x = 0 \\ -3x, & 0 < x < 2 \\ 2, & x \ge 2. \end{cases}$

For this function, find

(i) $\lim_{x\to -2^-} f(x)$,

(ii) $\lim_{x\to 0^-} f(x)$,

(iii) $\lim_{x\to -2^+} f(x)$,

(iv) $\lim_{x\to 0^+} f(x)$.

(4 marks)

(b) Evaluate the following limits

(i) $\lim_{x\to 8} \dfrac{x^{\frac{1}{3}}-2}{x-8}$,

(ii) $\lim_{x\to 3} \dfrac{\sin(x-2)}{x^2-5x+6}$.

(6 marks)

(c) Determine the value of d so that $f(x)=\begin{cases} 4x^2-1, & x<4, \\ 3dx & x \ge 4. \end{cases}$

is continuous for any value of x.

(4 marks)

(d) By using L'Hopital's rule, find the limits of the following expressions.

(i) $\lim_{x\to 0} \dfrac{e^x-1}{x^3}$,

(ii) $\lim_{x\to 0} \dfrac{\cos x+2x-1}{3x}$.

(6 marks)

Q2 (a) Find $f'(x)$ if $f(x)=e^{\ln(\cos 2x)}+\ln(x^2+2x-1)-\cos(3\pi x)$.

(4 marks)

(b) Determine constants A and B so that $P(x)=x^3+Ax^2+Bx+C$ has a critical points at $x=-1$ and $x=-3$.

(5 marks)

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(c) A curve is given by a parametric equation

$$x = t - \sin t \cos t \quad , \qquad y = 2\sin t .$$

By using parametric differentiation, find

(i) $\dfrac{dy}{dx}$,

(ii) $\dfrac{d^2y}{dx^2}$.

(6 marks)

(d) If $y = e^{-x}\ln(1+x)$, show that

$$(1+x)\frac{d^2y}{dx^2} + (2x+3)\frac{dy}{dx} + (x+2)y = 0.$$

(5 marks)

Q3 (a) If $x = 1$ is an initial approximation of the equation $x^3 + x^2 + x - 4 = 0$, find a better approximation.

(4 marks)

(b) Given that $f(x) = 9x^{\frac{2}{3}}$, write the expression of $f'(x)$. Find the value of x when $f(x) = 36$. Hence, find the approximate value of x when $f(x)$ increases from 36.0 to 36.2.

(5 marks)

(c) Find the equation of the tangent to the parabola $y = x^2 - 5x + 3$ at $x = 2$.

(5 marks)

(d) (i) Given $ad \neq bc$, show that the curve $y = \dfrac{ax+b}{cx+d}$ has no inflection.

(ii) The curve $y = \dfrac{x+5}{2x-3}$ intersects the x-axis at A. Find the gradient of the curve at A. Determine whether this curve has points of inflection or not. Hence, sketch the graph.

(6 marks)

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Q4 (a) If $I_n = \int x^n \sin x\,dx$, prove that

$$I_n = nx^{n-1}\sin x - x^n\cos x - n(n-1)I_{n-2}.$$

Hence, evaluate I_5.

(6 marks)

(b) Show that $\frac{d}{dx}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$. Hence evaluate

$$\int_0^{\frac{\pi}{4}} \sec^4 x\,dx.$$

(6 marks)

(c) By using the substitution $t = \tan x$, find

$$\int_0^{\frac{\pi}{4}} \frac{dx}{1+\cos 2x}.$$

(8 marks)

Q5 (a) Find the area of the region bounded by the curve $y = \tan x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{4}$.

(4 marks)

(b) Find the volume of the solid of revolutions when the region bounded by the lines $2y = x + 2$, the y-axis and $y = 5$ revolves 360° about y-axis.

(4 marks)

(c) If $y = \sqrt{1-x^2}\,\sin^{-1}(x)$, prove that

$$(1-x^2)\frac{dy}{dx} + xy = 1 - x^2.$$

(6 marks)

(d) Evaluate the following integrals.

(i) $\int \frac{dx}{1+16x^2}$ (ii) $\int \frac{\cos x}{\sin^2 x + 1}dx$

(6 marks)

- END OF QUESTIONS -

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Formulae

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \text{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \text{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \text{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x + y) + \cos(x - y)$	

Logarithm	Inverse Hiperbolic
$a^x = e^{x \ln a}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$, any x.
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$, $x \geq 1$
	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$, $-1 < x < 1$

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Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n\neq -1$
$\frac{d}{dx}\left[\ln\lvert x\rvert\right]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\,x]=-\text{cosec}\,x\cot x$	$\int \text{cosec}\,x\cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}\left[e^x\right]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\,x]=-\text{sech}\,x\tanh x$	$\int \text{sech}\,x\tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech}\,x]=-\text{cosech}\,x\coth x$	$\int \text{cosech}\,x\coth x\,dx = -\text{cosech}\,x + C$

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Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\lvert a\rvert\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0.$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a$
$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a.$
$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a}\text{cosech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a.$

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y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}, \quad \lvert u\rvert < 1.$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}, \quad \lvert u\rvert < 1.$
$\tan^{-1} u$	$\frac{1}{1+u^2}.\frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2}.\frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}.\frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\text{cosec}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}}.\frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}}.\frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}.\frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\tanh^{-1} u$	$\frac{1}{1-u^2}.\frac{du}{dx}, \quad \lvert u\rvert < 1.$
$\coth^{-1} u$	$-\frac{1}{1-u^2}.\frac{du}{dx}, \quad \lvert u\rvert > 1.$
$\text{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}}.\frac{du}{dx}, \quad 0 < u < 1.$
$\text{cosech}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{1+u^2}}.\frac{du}{dx}, \quad u \neq 0.$

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