UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	CALCULUS
COURSE CODE	:	BWC10303
PROGRAMME	:	BWC
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) (i) Give a definition for continuity.

(2 marks)

(ii) The graph of $f(x)$ is given in **Figure Q1(a)(ii)**. Based on this graph, determine where the function is discontinuous. Give reasons for your choice.

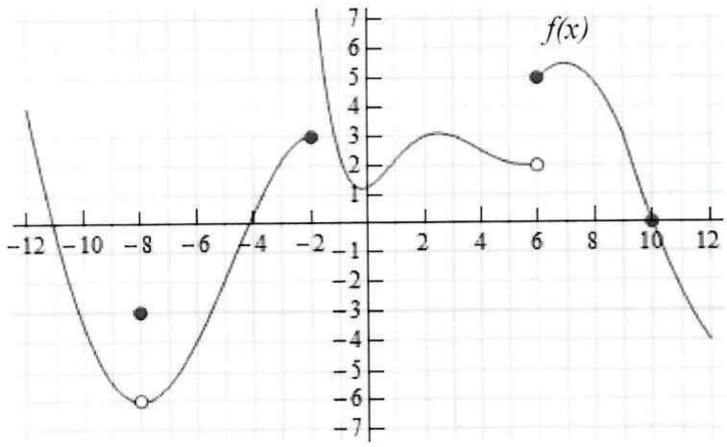

Figure Q1(a)(ii),

(6 marks)

(b) Given

$$f(x)=\frac{ax+bx^2}{4x-8x^2}.$$

Find the value of

(i) a if $\lim_{x\to 0} f(x)=2$.

(ii) b if $\lim_{x\to\infty} f(x)=1$.

(8 marks)

(c) Find

(i) $\lim_{x\to 0}\frac{x+5}{x^2-25}$.

(ii) $\lim_{x\to -\infty}\frac{4x^2-x}{2x^3-5}$.

(iii) $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$.

(9 marks)

Q2 (a) By using the chain rule, find the derivatives of the following functions.

(i) $y = \sin\left(x^2 + 2x - 1\right)$

(ii) $y = \dfrac{1}{x^3 + 2x - 3}$

(12 marks)

(b) (i) Find the slopes of the curve $y^2 - x + 1 = 0$ at the points (2 , -1) and (2 , 1)

(ii) Find $\dfrac{dy}{dx}$ when $x = t^2$, $y = t + \dfrac{1}{t}$.

(13 marks)

Q3 (a) Find the following integrals

(i) $\int \dfrac{2x+4}{x^3 + 2x^2}\, dx$.

(ii) $\int_0^1 e^{\ln(x+1)^2}\, dx$.

(12 marks)

(b) Use the method of integration by parts to find $\int_0^{\pi/2} x \sin 4x \, dx$.

(7 marks)

(c) Evaluate $\int_0^1 (x^3 + 2x)e^{2x}\, dx$ by using suitable method.

(6 marks)

Q4 (a) Find $\int \dfrac{3}{\sqrt{x^2 - 9}}\, dx$ by using a hyperbolic substitution.

(7 marks)

(b) Determine the area of the region bounded by $y = x^2 + 2$, $y = \sin(x)$ $x = -1$ and $x = 2$.

(8 marks)

(c) Given $y^2 - 4x^2 = 9$.

(i) Find the curvature κ of the given curve at $x = 2$.

(ii) Find the radius of curvature ρ of the given curve at $x = 1$.

(10 marks)

- END OF QUESTIONS –

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \text{sech}^2 x \, dx = \tanh x + C$$

$$\int \text{csch}^2 x \, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$

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Formulae

$\sqrt{k^2 - x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$
$$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$
$$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$$
$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

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CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$