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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	NUMERICAL METHOD I / NUMERICAL ANALYSIS I
COURSE CODE	:	BWA 21303 / BWA 20903
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) ALL CALCULATIONS MUST BE IN **THREE (3) DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) State the differences between round-off error and truncation error. Give an example for each.

(6 marks)

(b) Given a nonlinear function $f(x)=\frac{1}{x}-a$, where a is a constant.

(i) Show that the Newton-Raphson iteration scheme for the function above is given by

$$x_{k+1}=x_k\left(2-ax_k\right).$$

(4 marks)

(ii) Hence, find the root of $f(x)$ if $a=3$ and start with $x_0=0.3$.

(3 marks)

(c) A biologist has placed three strains of bacteria (denoted I, II and III) in a test tube, where they will feed with three different food sources (A, B and C). Each day 700 units of A, 400 units of B and 500 units of C are placed in the test tube. Each bacteria consumes a certain number of units of each food per day, as shown in **Table Q1(c)** below.

Table Q1(c)

	Bacteria Strain I	Bacteria Strain II	Bacteria Strain III
Food A	0	1	2
Food B	5	1	0
Food C	1	3	1

(i) Form a system of linear equations based on the above problem.

(4 marks)

(ii) Hence, determine the number of bacteria of each strain that can coexist in the test tube and consume all of the food by using Gauss-Seidel iteration method.

(8 marks)

Q2 (a) Construct the natural cubic spline $S(x)$ using the following data given in **Table Q2(a).**

Table Q2(a)

x_i	-2	-1	0
$f(x_i)$	1.4	2.9	2.2

(12 marks)

(b) By expanding $f(x+h)$ in a Taylor series up to three terms, deduce an expression for the truncation error e^T in the first derivative 2-point forward difference formula,

$$f'(x) = \frac{f(x+h) - f(x)}{h} + e^T .$$

(4 marks)

(c) A point P is moving along the curve whose equation is $y = \sqrt{t^3 + 17}$. Calculate the velocity and the acceleration of P at $t = 3.35$ s, with $h = 0.05$ using

(i) 3-point central difference formula, and

(5 marks)

(ii) 5-point difference formula.

(4 marks)

Q3 (a) Consider a symmetric matrix

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}.$$

(i) Determine the interval of which the eigenvalues of matrix A above are contained by using Gerschgorin's theorem.

(7 marks)

(ii) Find the dominant eigenvalue and corresponding eigenvector for matrix A by using power method with $v^{(0)} = (1 \quad 1 \quad 0)^T$ and $\varepsilon = 0.005$.

(5 marks)

(iii) Hence, find the smallest eigenvalue (in absolute value) and corresponding eigenvector for matrix A by using shifted power method with $v^{(0)} = (1 \quad 1 \quad 1)^T$ and $\varepsilon = 0.005$.

(6 marks)

(b) A basketball player makes a successful shot from the free throw line. Suppose that the path of the ball from the moment of release to the moment it enters the hoop is described by

$$y = 2.15 + 2.09x - 0.41x^2, \qquad 0 \le x \le 3.6$$

where x is the horizontal distance (in meters) from the point of release, and y is the vertical distance (in meters) above the floor. Approximate the distance of the ball travels from the moment of release to the moment it enters the hoop, by using the appropriate Simpson's rule with $h = 0.4$.

[Hint: Arc length of the curve, $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$]

(7 marks)

Q4 (a) Given an initial value problem

$$y' = 1.5x^{1/3}, \quad y(0) = 0.$$

(i) Show that Euler's method fails to approximate the solution of the problem and justify your answer.

(5 marks)

(ii) Suggest one way to make it possible to solve the initial value problem.

(1 mark)

(b) Consider an ordinary differential equation

$$(1 + x^2)\frac{dy}{dx} - xy = 0 \text{ over } x = 2(0.25)2.5,$$

with initial condition $y(2) = 5$.

(i) Solve the initial value problem using fourth-order Runge-Kutta method.

(6 marks)

(ii) Hence, calculate the absolute error for each approximation if the exact solution for the differential equation is given by $y(x) = \sqrt{5(1 + x^2)}$.

(3 marks)

(c) Given a boundary-value problem,

$$y'' + xy = x^3 - \frac{4}{x}, \quad 1 \le x \le 2,$$

with boundary conditions, $4y(1) + y'(1) = 0$, and $3y(2) + 2y'(2) = 0$. Derive a system of linear equations in matrix-vector form for the problem with $h = \Delta x = 0.2$ using finite-difference method. (Note: Do not solve the problem)

(10 marks)

- END OF QUESTIONS -

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FORMULA

Nonlinear equations

Newton-Raphson method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \ldots$

System of linear equations

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, \quad \forall i = 1, 2, 3, \ldots, n$

Interpolation

Cubic spline:

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k)$$

where $k = 0,1,2,\ldots,n-1$

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, \quad k = 0,1,2,\ldots,n-1$$

$b_k = 6(d_{k+1} - d_k), \quad k = 0,1,2,\ldots,n-2$

$m_0 = 0$

$m_n = 0$

$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, \quad k = 0,1,2,\ldots,n-2$

Eigen value

Gerschgorin's theorem:

$$r_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|, \qquad D_i = \{z \in \mathbf{C} : |z - a_{ii}| \le r_i\}, \qquad \lambda_k \in \bigcup_{i=1}^{n} D_i \quad \text{for } k = 1,2,\ldots,n$$

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Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0, 1, 2, ...$

Shifted Power Method: $\mathbf{A}_{\text{shifted}} = \mathbf{A} - l_{\text{Largest}}\mathbf{I}, \ \lambda_{\text{smallest}} = \lambda_{\text{Shifted}} + l_{\text{Largest}}$

Numerical differentiation and integration

Differentiation:

First derivatives:

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

Second derivatives:

3-point central difference: $f''(x) \approx \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference: $f''(x) \approx \dfrac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

Integration:

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \dfrac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3}) \right]$$

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Ordinary differential equations

Initial value problems:

Euler's Method: $y_{i+1} = y_i + h\,f(x_i, y_i)$

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i,\, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2},\, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2},\, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h,\, y_i + k_3)$

Boundary value problems:

Finite difference method: $y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$, $\qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$