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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	OPTIMIZATION TECHNIQUES II
COURSE CODE	:	BWA 40703
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

TERBUKA

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 Consider the following problem

$$\text{Minimize} \quad x_1^2 + 2x_2^2$$

subject to

$$1 - x_1 - x_2 \leq 0 .$$

(a) Define the penalty function.

(2 marks)

(b) Derive the first-order necessary conditions.

(4 marks)

(c) Show that for $c > 0$,

$$x_1^* = \frac{2c}{2+3c} \text{ and } x_2^* = \frac{c}{2+3c}$$

(9 marks)

(d) Calculate the value of x_1^* and x_2^* for $c = 0.1, 1.0, 10, 100, 1000$.

(5 marks)

Q2 The barrier objective function is defined by

$$r(\mathbf{x}, \mu) = 2x_1^2 + 9x_2 - \mu\left(\frac{1}{4 - x_1 - x_2}\right).$$

(a) Find the gradient of $r(\mathbf{x}, \mu)$.

(4 marks)

(b) Show that $x_1^* = 2.25$ and $x_2^* = 1.75 + 0.333\sqrt{\mu}$ for $\mu > 0$.

(10 marks)

(c) Verify that the objective function $f(\mathbf{x}^*)$ is 25.857.

(6 marks)

Q3 Consider the general problem

$$\text{Minimize } f(\mathbf{x})$$
$$\text{subject to}$$
$$h(\mathbf{x}) = \mathbf{0}$$

with $\mathbf{x} \in \Re^n$, $h(\mathbf{x}) \in \Re^m$, and $m < n$.

(a) Define the penalty objective function $q(\mathbf{x})$ by applying the standard quadratic penalty method.

(2 marks)

(b) Find the gradient and the Hessian for the penalty objective function $q(\mathbf{x})$ defined in (a).

(6 marks)

(c) Illustrate that for $\alpha_k > 0$ and $c > 0$,

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k[\mathbf{I} + c\nabla\mathbf{h}(\mathbf{x}_k)^{\mathrm{T}}\nabla\mathbf{h}(\mathbf{x})]^{-1}\nabla q(\mathbf{x}_k)^{\mathrm{T}}$$

using $(\mathbf{I} + \mathbf{Q}(\mathbf{x}_k))\mathbf{d}_k = \nabla q(\mathbf{x}_k)$ and assuming $\mathbf{L}(\mathbf{x}_k)\mathbf{d}_k = \mathbf{0}$.

(10 marks)

(d) State **TWO (2)** properties of $[\mathbf{I} + c\nabla\mathbf{h}(\mathbf{x}_k)^{\mathrm{T}}\nabla\mathbf{h}(\mathbf{x})]$.

(2 marks)

Q4 Assuming that $\mathbf{x}^*$ is a regular point, then there will be a corresponding Lagrange multiplier vector $\boldsymbol{\lambda}^*$ such that

$$\nabla f(\mathbf{x}^*) + (\boldsymbol{\lambda}^*)^{\mathrm{T}} \nabla \mathbf{h}(\mathbf{x}^*) = \mathbf{0},$$

and the Hessian of the Lagrangian

$$\mathbf{L}(\mathbf{x}^*) = \mathbf{F}(\mathbf{x}^*) + (\boldsymbol{\lambda}^*)^{\mathrm{T}} \mathbf{H}(\mathbf{x}^*)$$

must be positive semidefinite on the tangent subspace

$$M = \{\mathbf{x} : \nabla \mathbf{h}(\mathbf{x}^*) \cdot \mathbf{x} = \mathbf{0}\}.$$

(a) Show that the dual function ϕ has the gradient

$$\nabla \phi(\boldsymbol{\lambda}) = \mathbf{h}(\mathbf{x}(\boldsymbol{\lambda}))^{\mathrm{T}}.$$

(9 marks)

(b) Determine that the Hessian of the dual function is

$$\Phi(\boldsymbol{\lambda}) = -\nabla \mathbf{h}(\mathbf{x}(\boldsymbol{\lambda}))\mathbf{L}^{-1}(\mathbf{x}(\boldsymbol{\lambda}), \boldsymbol{\lambda}) \nabla \mathbf{h}(\mathbf{x}(\boldsymbol{\lambda}))^{\mathrm{T}}.$$

(11 marks)

Q5 Consider the simple quadratic problem

$$\text{Minimize} \quad 2x_1^2 + 2x_1x_2 + x_2^2 - 2x_2$$

subject to

$$x_1 = 0.$$

(a) Define the augmented Lagrangian function.

(2 marks)

(b) Deduce that

$$x_1^* = -\frac{(2+\lambda)}{(2+c)} \text{ and } x_2^* = \frac{(4+c+\lambda)}{(2+c)}.$$

(11 marks)

(c) Outline that the iterative process for λ_k is given by

$$\lambda_{k+1} = \left(\frac{2}{2+c}\right)\lambda_k - \frac{2c}{2+c}.$$

(7 marks)

– END OF QUESTIONS –