UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWA 20303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) (i) State the difference between a particular solution and a general solution of a linear differential equation.

(2 marks)

(ii) Determine the order, the unknown function and the independent variable of the following differential equation.

$$y\left(\frac{d^2x}{dy^2}\right)^3 = y^2 + 1.$$

(3 marks)

(b) Solve the first order differential equation given as

$$(xy^3)\frac{dy}{dx} = 2y^4 + x^4.$$

(8 marks)

(c) A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there are 50 milligrams of the material present and after two hours it is observed that the material has lost 10 percent of its original mass, find

(i) an expression for the mass of the material at any time, t.

(5 marks)

(ii) the mass of the material after five hours.

(2 marks)

Q2 (a) By the undetermined coefficient method, find the general solution of the differential equation,

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 64e^{-t}.$$

(10 marks)

(b) Solve the differential equation,

$$y'' + 2y' + y = e^{-x}\ln x$$

by using the variation of parameters method.

(10 marks)

Q3 (a) Verify that the Laplace transform of

$$f(t) = \sinh\frac{1}{2}t + \cosh 3t$$

is given by

$$\frac{4s^3 + 2s^2 - s - 18}{(4s^2 - 1)(s^2 - 9)}.$$

(5 marks)

(b) Determine the inverse Laplace transform of $\dfrac{2}{s(s^2+1)}$.

(6 marks)

(c) Solve the following differential equation by using the Laplace transform.

$$y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 0, \text{ and } f(t) = \begin{cases} 0, & t < 1 \\ 2, & t \geq 1 \end{cases}$$

(9 marks)

Q4 (a) By using the power series method, verify that $y = c_0 \cos x + c_1 \sin x$ is the solution of $y'' + y = 0$.

Assume that the solution has the form of $y = \sum_{m=0}^{\infty} c_m x^m$.

(10 marks)

(b) By using an appropriate power series method, determine the solution to the given equation up to x^4 only.

$$y'' = -2xy, \quad y(2) = 1, \quad y'(2) = 0.$$

(10 marks)

Q5 (a) Given the differential equation,

$$x^{(3)} - 2x'' + x = 0.$$

(i) Reduce the equation above to a system of first order differential equations.
(5 marks)

(ii) Write the system in **Q5** (a) (i) in matrix form.
(3 marks)

(b) By using the Laplace transform, solve the following system of linear differential equations

$$\begin{aligned} y'' + z + y &= 0 \\ z' + y' &= 0 \end{aligned}$$

subject to conditions $y(0) = 0, \quad y'(0) = 1, \quad z(0) = 1.$
(12 marks)

– END OF QUESTIONS –

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$ or $a\ddot{y}+b\dot{y}+cy=0$ or $a\dfrac{d^2y}{dx^2}+b\dfrac{dy}{dx}+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay''+by'+cy=f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x)=A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c=Ay_1+By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y=y_c+y_p, \quad \text{and} \quad y_p=uy_1+vy_2,$$

$$\text{where } u=-\int\frac{y_2f(x)}{aW}dx, \qquad v=\int\frac{y_1f(x)}{aW}dx \quad \text{and} \quad W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}=y_1y_2'-y_2y_1'.$$

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Representation of Functions in Power Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.. \ , \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots.. \ , \quad e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots..$$

Maclaurin series

$$y(x) = \sum_{m=0}^{\infty} y^m(0)\frac{x^m}{m!} = y(0) + y'(0)x + \frac{1}{2!}y''(0)x^2 + \frac{1}{3!}y'''(0)x^3 + \ldots\ldots\ldots.$$

Taylor series, at $x = a$,

$$y(x) = \sum_{m=0}^{\infty} y^m(a)\frac{(x-a)^m}{m!} = y(a) + y'(a)(x-a) + \frac{1}{2!}y''(a)(x-a)^2 + \frac{1}{3!}y'''(a)(x-a)^3 + \ldots\ldots\ldots.$$