UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	PARTIAL DIFFERENTIAL EQUATION
COURSE CODE	:	BWA 30303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Solve the equation $u_x + 2xy^2 u_y = 0$. Sketch some of the characteristic curves.

(4 marks)

(b) Use the coordinate method to solve the equation

$$u_x + 2u_y + (2x - y)u = 0.$$

(7 marks)

(c) Solve the initial value problem for the transport equation with damping

$$u_t - 4u_x + u = 0,\ \ t > 0,\ \ x \in \mathbb{R},\ \ u(0,x) = e^{-x^2}.$$

Sketch the graph of the solution at $t = 3$.

(Hint: Multiply by e^t and consider the problem solved by $v = e^t u$)

(8 marks)

Q2 (a) What is the type of the equation

$$u_{xx} - 4u_{xy} + 4u_{yy} = 0?$$

Show by direct substituition that $u(x,y) = f(y+2x) + xg(y+2x)$ is a solution for arbitrary functions f and g.

(5 marks)

(b) Reduce the elliptic equation

$$u_{xx} + 3u_{yy} - 2u_x + 24u_y + 5u = 0$$

to the form $v_{xx} + v_{yy} + cv = 0$ by a change of dependent variables $u = ve^{\alpha x + \beta y}$ and then a change of scale $x = \mu\zeta,\ y = \gamma\eta$ where μ and γ are constants.

(12 marks)

(c) Construct and sketch the graph of the even and odd $2\pi-$periodic extensions of the function $f(x) = 1 - x$. What are their Fourier series? Discuss convergence of each.

(13 marks)

Q3 (a) Find a formal solution of a vibrating string with fixed ends:

$$u_{tt} - c^2 u_{xx} = 0, \qquad 0 < x < L, \ t > 0$$
$$u(0,t) = u(L,t) = 0, \qquad t \geq 0$$
$$u(x,0) = f(x), \ u_t(x,0) = g(x), \ 0 \leq x \leq L$$

using the separation of variables method.

(12 marks)

(b) Prove that the solution in **Q3 (a)** can be represented as a superposition of a forward and a backward wave.

(4 marks)

Q4 Given the heat equation

$$u_t = k u_{xx} \text{ for } 0 < x < L, \ t > 0,$$
$$u(0,t) = u_x(L,t) = 0 \text{ for } t > 0,$$
$$u(x,0) = f(x) \text{ for } 0 < x < L,$$

describe heat conduction in a bar of length L with the left end kept at temperature zero but with an insulation condition on the right end. Using separation of variables show that the problem to solve for X is

$$X'' - pX = 0; \ X(0) = X'(L) = 0.$$

By considering case on p, show that this problem has eigenvalues

$$p_n = -\frac{(2n-1)^2 \pi^2}{4L^2}$$

for $n = 1, 2, \ldots$. Show that for $n = 1, 2, \ldots$, the functions

$$u_n(x,t) = b_n \sin\left(\frac{(2n-1)\pi x}{2L}\right) e^{-(2n-1)^2 \pi^2 kt / 4L^2}$$

are solutions of the heat equation satisfying both boundary conditions. To satisfy the initial condition, attempt a superposition

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{(2n-1)\pi x}{2L}\right) e^{-(2n-1)^2 \pi^2 kt / 4L^2}.$$

Require that

$$u(x,0) = f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{(2n-1)\pi x}{2L}\right).$$

Derive a formula for the b_n's by reasoning informally as in the problem with one radiating end.

(15 marks)

Q5 The temperature distribution, $u(r,\theta)$ in a circular metal disc of radius 1 that has its top and bottom insulated is described by equation

$$u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0,\quad 0\le r<1,\quad 0<\theta<2\pi.$$

(a) Show that the general solution of Laplace equation is

$$u(r,\theta)=\frac{A_0}{2}+\sum_{n=1}^{\infty} r^n\left[A_n\cos(n\theta)+B_n\sin(n\theta)\right].$$

(15 marks)

(b) Given

$$u_r(1,\theta)=\begin{cases}1, & 0<\theta<\pi,\\ -1, & \pi<\theta<2\pi.\end{cases}$$

Show that the solution of Laplace equation is

$$u(r,\theta)=\frac{A_0}{2}+\sum_{n=1}^{\infty} r^n\left(\frac{1}{\pi n}\right)\left[-\frac{2}{n}\cos(n\pi)+\frac{2}{n}\right]\sin(n\theta).$$

(5 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

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Formulae

Fourier Series: $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left\{a_n \cos\left(\frac{n\pi x}{\ell}\right) + b_n \sin\left(\frac{n\pi x}{\ell}\right)\right\}$,

where $a_0 = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)dx$,

$$a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx, \quad n = 1,2,3,\ldots,$$

$$b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx, \quad n = 1,2,3,\ldots$$

Half Range Cosine Series: $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{\ell}\right)$,

where $a_0 = \frac{2}{\ell}\int_{0}^{\ell} f(x)dx$,

$$a_n = \frac{2}{\ell}\int_{0}^{\ell} f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx, \quad n = 1,2,3,\ldots$$

Half Range Sine Series: $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right)$,

where $b_n = \frac{2}{\ell}\int_{0}^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx, \quad n = 1,2,3,\ldots$