CONFIDENTIAL

UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ACTUARIAL MATHEMATICS II
COURSE CODE	:	BWA 31503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

CONFIDENTIAL

Q1 (a) You are given the Illustrative Life Table **(Table Q1 (a))** with interest rate of 6%. A 40-year-old male purchased the 10-year endowment insurance where the net level premium is RM 81.36 and the death benefit is RM 1,000. Determine the terminal reserve for the fourth year of a 40-year old male.

Table Q1 (a): Illustrative Life Table

Age	Number Alive	Number That Die
40	937.72	2.83
41	934.89	3.08
42	931.81	3.32
43	928.50	3.59
44	924.90	3.88

(10 marks)

(b) Cash value is the amount available in cash upon cancellation of an insurance policy. Identify **THREE (3)** common insurance options that use the policy's net cash values and discuss the similarities or differences among these three insurance options.

(10 marks)

Q2 (a) Consider an insurance portfolio that will produce zero, one, two, or three claims in a fixed time period with probabilities 0.1, 0.3, 0.4, and 0.2, respectively. An individual claim will be of amount 1, 2, or 3 with probabilities 0.5, 0.4, and 0.1 respectively.

(i) Construct a table and compute $f_s(x) = \Pr(S = s)$ for $x = 0,1,2,3,4$.

(10 marks)

(ii) Find $E[N]$, $Var(N)$ and $E[X]$.

(4 marks)

(b) Suppose that the claim amount distribution is the same as in **Q2 (a).** The distribution of N follows Poisson distribution. The formula for the expectation of S given by

$$E[S] = \lambda E[X] = \lambda p_1,$$

and the variance of S

$$Var(S) = \lambda E[X^2] = \lambda p_2.$$

Use these formulas and $E[N]$ from **Q2(a)(ii)** to compute $E[S]$ and $Var(S)$.

(6 marks)

Q3 (a) The actuarial present value of the family income benefit is given by

$$E(Z)=\int_0^n v^t \bar{a}_{\overline{n-t|}}\ {}_tp_x\ \mu_x(t)\,dt$$

This integral can be converted to a current payment integral by integration by parts,

$$\bar{a}_{\overline{n|}}-\int_0^n v^t\ {}_tp_x dt=\int_0^n v^t\left(1-{}_xp_x\right)dt=\bar{a}_{\overline{n|}}-\bar{a}_{x:\overline{n|}}$$

A policy provides a continuous annuity-certain of 1 per annum beginning at the date of death of age 35. You are given the following information:

- In the event of death prior to age 65, a family income benefit ceasing at age 65, and
- In the event of survival to age 65, a life annuity with 5 years certain.

(i) Construct a table that gives the conditions required for payments at time t and the corresponding probabilities.

(6 marks)

(ii) Calculate the actuarial present value of the benefits.

(4 marks)

(b) Consider a policy issued at age 35 with an initial gross premium of 1,000 and an initial benefit of 120,000. The policyholder wishes to change the premium of the policy after 5 years to 1,500 and the benefit amount to 150,000. Use the Illustrative Life Table (**Table Q3(b)**) with 6% interest to determine the reserve after 10 years of original issue if the fifth year reserve is 3,321.25.

Table Q3(b): Illustrative Life Table

Age	l_x	d_x	$1{,}000q_x$
30	95 013.79	145.2682	1.5289
31	94 868.53	152.6317	1.6089
32	94 715.89	160.6896	1.6965
33	94 555.20	169.5052	1.7927
34	94 385.70	179.1475	1.8980
35	94 206.55	189.6914	2.0136
36	94 016.86	201.2179	2.1402
37	93 815.64	213.8149	2.2791
38	93 601.83	227.5775	2.4313
39	93 374.25	242.6085	2.5982
40	93 131.64	259.0186	2.7812
41	92 872.62	276.9271	2.9818
42	92 595.70	296.4623	3.2017
43	92 299.23	317.7619	3.4427
44	91 981.47	340.9730	3.7070
45	91 640.50	366.2529	3.9966

(10 marks)

Q4 (a) A continuous annuity is payable as long as any of (w), (x), (y) and (z) is alive. The payment rate starts at 8 and is reduced by 50% for each death.

(i) Express the actuarial present value for such an annuity in terms of $\bar{a}_{\overline{wxyz}}^{[k]},\ k=1,2,3,4$.

(4 marks)

(ii) Prepare a difference table and express the actuarial present value for the annuity in terms of actuarial present values for annuities on single- and joint-life statuses.

(10 marks)

(b) The symbol ${}_nq_{\underset{1}{x}yz}^{\;\;3}$ refer to events in which $T(x)<T(y)<T(z)$. By conditioning on each of the future lifetimes, write three different integrals for ${}_nq_{\underset{1}{x}yz}^{\;\;3}$.

(6 marks)

Q5 (a) From **Figure Q5(a)**, use the double integral method to show that the number of lives that will attain age x_0 between times t_0 and t_0+1 and die before time t_0+3 is given by

$$\int_{t_0}^{t_0+1} l(x_0, y-x_0)\,dy - \int_{x_0+2}^{x_0+3} l(w, t_0+3-w)\,dw.$$

The generation force of mortality at age x for those born at time u is denoted by

$$\mu(x,u) = -\frac{1}{l(x,u)}\frac{\partial}{\partial x}l(x,u).$$

(10 marks)

(b) A population density function is defined by

$$l(x,u)=b(u)s(x,u).$$

Then, let

$$b(u)=100\left[1-e^{-u/100}\right] \quad u>0,$$

and

$$s(x)=e^{-x/100} \quad x>0.$$

Calculate the number of lives who will attain age 25 between times 50 and 51 and die before time 53.

(10 marks)

-END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020 PROGRAMME CODE : BWA
COURSE NAME : ACTUARIAL MATHEMATICS II COURSE CODE : BWA 31503

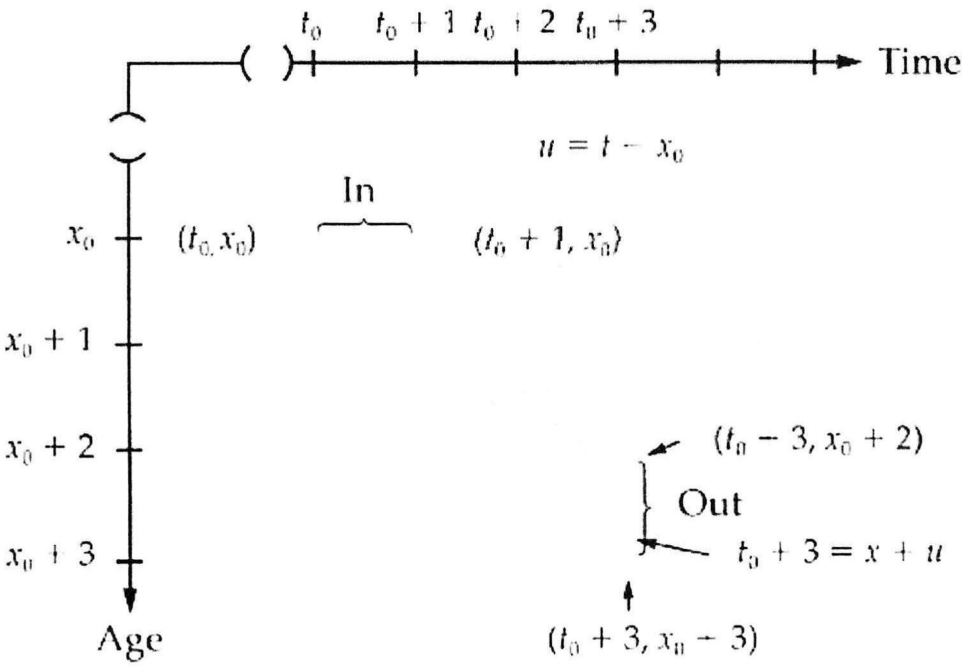

Figure Q5(a)

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2019/2020
COURSE NAME : ACTUARIAL MATHEMATICS II
PROGRAMME CODE : BWA
COURSE CODE : BWA 31503

FORMULAE

$$\Pr(N=n)=\frac{\lambda^n e^{-\lambda}}{n!}$$

$$\ddot{a}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^k\cdot {}_kp_x$$

$$A^{1}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^{k+1}\cdot {}_kp_x\cdot q_{x+k}$$

$${}_nE_x=v^n\,{}_nP_x$$

$$v^n=\frac{1}{(1+i)^n}$$

$${}_kP_x=\frac{l_{40+k}}{l_{40}}$$

$$-{}_0V=P-\upsilon q_x b$$

$${}_kV=\frac{{}_0V+P\ddot{a}_{x:\overline{k}|}-bA^{1}_{x:\overline{k}|}}{{}_kE_x}$$

$${}_{k+g}V'=\frac{{}_kV'+P'\ddot{a}_{x+k:\overline{g}|}-b'A^{1}_{x+k:\overline{g}|}}{{}_gE_{x+k}}$$

$$\int_{t_0}^{t_1} l(x,t-x)\,dt$$

$$\int_{x_0}^{x_1} l(x,t_0-x)\,dx$$