UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	RISK THEORY
COURSE CODE	:	BWA 40803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) An individual faces the following possible losses as in **Table Q1(a)**:

Table Q1(a)

Loss Size	*Probability*
RM1000	0.001
100	0.100
0	0.899

The utility function of a potential purchaser of insurance is given:

$$u(x) = x^{0.6}.$$

(i) Justify that this person is risk averse.

(7 marks)

(ii) Compute the expected value of loss and the maximum premium of this individual would pay for insurance given the above loss distribution and initial wealth of RM2000.

(9 marks)

(b) An insurer has offered an individual insurance cover against a random loss, X, where X has a mixed distribution with probability 0.8 and the probability distribution function f given by

$$f(x) = 0.2(e^{-0.01x}) \quad \text{for} \quad x \geq 0.$$

Calculate the minimum premium that the insurer would accept if the insurer bases decisions on the utility function

$$u(x) = -e^{-0.005x}.$$

(9 marks)

Q2 (a) Consider a group life insurance contract with an accidental death benefit. Assume that for all members the probability of death in the next year is 0.01 and that 30% of deaths are accidental. For 50 employees the benefit for an ordinary death is 50,000 and for an accidental death it is 100,000. For the remaining 25 employees the benefits are 75,000 and 150,000, respectively. Develop an individual risk model and determine its mean and variance.

(7 marks)

(b) The probability density function of aggregate claims, S, is given by $f_S(x) = 3x^{-4}$, $x \geq 1$. The relative loading θ and the value λ are selected so that

$$\mathbb{P}[S \leq (1+\theta)\mathbf{E}(S)] = \mathbb{P}\left[S \leq \mathbf{E}(S) + \lambda\sqrt{Var(S)}\right] = 0.90.$$

Calculate λ and θ.

(9 marks)

(c) An insurance company sells hospitalisation reimbursement insurance. The benefit payment for a standard hospital stay follows a lognormal distribution with mean 7 and standard deviation 2 but 25% of all hospitalizations are for accidentally causes. However, the benefit payment for a hospital stay due to an accident is twice as much as a standard benefit. Calculate the probability that a benefit payment will exceed RM15000 by using normal approximation.

(9 marks)

Q3 (a) A corporation provides health insurance to its 100 employees, and has purchased stop-loss reinsurance with a deductible of 330,000 to limit its liability for health care payments. The number of medical care visits per employee per year has a Poisson distribution with parameter $\lambda = 6$. The cost of each visit has an exponential distribution with mean 500. The number of visits and costs per employee are independent of the experience of the other employees. Using the normal distribution to approximate the distribution of annual aggregate claims, calculate the probability that the reinsurer will make a payment to the corporation at year end. [Hint: that aggregate claim will exceed the deductible of 330,000]

(10 marks)

(b) Two types of claims are made to an insurance company. The number of Type A claims follows a Poisson distribution with $\lambda = 12$, and the amount of a Type A claim is uniformly distributed over $(0,1)$. The number of Type B claims follows a Poisson distribution with $\lambda = 4$ and the amount of a Type B claim is uniformly distributed over $(0,5)$. All claim numbers and claim amounts are mutually independent. Use the normal approximation to determine the probability that the total of claim amounts exceeds 18.

(15 marks)

Q4 (a) An insurance fund pays an annual claim at the end of each year of possible amounts 3, 5, or 7, with probabilities $p(3) = 0.75$, $p(5) = 0.15$ and $p(7) = 0.10$. The fund collects a premium at the beginning of each year equal to the expected claim plus a relative security loading of 30%. Expenses and interest can be ignored. If the fund starts with an initial surplus of 3, compute the probability of ruin within the first two years, which we have denoted by $\psi(3,2)$.

(12 marks)

(b) Suppose that the relative security loading is $\theta = 2$ and the gamma distribution has $\alpha = 2$. To avoid confusion, let β be the scale parameter for the gamma distribution. Determine the adjustment coefficient and its ruin probability.

(13 marks)

- END OF QUESTIONS -

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Probability Distributions

Discrete Distributions	p.f.	Restrictions on Parameters	Moment Generating Function, $M(s)$	Moments: Mean	Moments: Variance
Binomial	$\binom{n}{x}p^x q^{n-x}$, $x = 0, 1, \ldots n$	$0 < p < 1$, $q = 1 - p$	$(pe^s + q)^n$	np	npq
Bernoulli	Special case $n = 1$				
Negative Binomial	$\binom{r+x-1}{x}p^r q^x$, $x = 0, 1, 2, \ldots$	$0 < p < 1$, $q = 1 - p$, $r > 0$	$\left(\frac{p}{1 - qe^s}\right)^r$, $qe^s < 1$	$\frac{rq}{p}$	$\frac{rq}{p^2}$
Geometric	Special case $r = 1$				
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$	$\lambda > 0$	$e^{\lambda(e^s - 1)}$	λ	λ
Uniform	$\frac{1}{n}$, $x = 1, \ldots, n$	n, a positive integer	$\frac{e^s(1 - e^{sn})}{n(1 - e^s)}$, $s \neq 0$; $1, s = 0$	$\frac{n + 1}{2}$	$\frac{n^2 - 1}{12}$

Continuous Distributions	p.d.f.	Restrictions on Parameters	Moment Generating Function, $M(s)$	Moments: Mean	Moments: Variance
Uniform	$\frac{1}{b - a}$, $a < x < b$	—	$\frac{e^{bs} - e^{as}}{(b - a)s}$, $s \neq 0$; $1, s = 0$	$\frac{b + a}{2}$	$\frac{(b - a)^2}{12}$
Normal	$\frac{1}{\sigma\sqrt{2\pi}}\exp[-(x - \mu)^2/2\sigma^2]$, $-\infty < x < \infty$	$\sigma > 0$	$\exp(\mu s + \sigma^2 s^2/2)$	μ	σ^2
Gamma	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$, $x > 0$	$\alpha > 0, \beta > 0$	$\left(\frac{\beta}{\beta - s}\right)^\alpha$, $s < \beta$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Exponential	Special case $\alpha = 1$				
Chi-square	Special case $\alpha = \frac{k}{2}, \beta = \frac{1}{2}$	k, a positive integer			
Inverse Gaussian	$\frac{\alpha}{\sqrt{2\pi\beta}}x^{-3/2}\exp\left[-\frac{(\beta x - \alpha)^2}{2\beta x}\right]$, $x > 0$	$\alpha > 0, \beta > 0$	$\exp\left[\alpha\left(1 - \sqrt{1 - \frac{2s}{\beta}}\right)\right]$, $s < \frac{\beta}{2}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Pareto	$\alpha x_0^\alpha / x^{\alpha+1}$, $x > x_0$	$x_0 > 0, \alpha > 0$		$\frac{\alpha x_0}{\alpha - 1}$, $\alpha > 1$	$\frac{\alpha x_0^2}{(\alpha - 2)(\alpha - 1)^2}$, $\alpha > 2$
Lognormal	$\frac{1}{x\sigma\sqrt{2\pi}}\exp[-(\log x - m)^2/2\sigma^2]$, $x > 0$	$-\infty < m < \infty$, $\sigma > 0$		$e^{m+\sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2m+\sigma^2}$

Utility Theory

- An insurer with utility $u(\cdot)$ and wealth w needs π or more to cover X if $E[u(w = \pi - X)] = u(w)$.
- Expected value: $E[w] = pw_1 + qw_2$
- Expected utility function: $E[u(w)] = pu(w_1) + qu(w_2)$

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Individual Risk Model

- Aggregate claim is $S = X_1 + X_2 + \cdots + X_n$ where n is number of risk unit insured and X_i is the distribution of amount of claims
- X, the claim random variable. Its p.f is

$$f_X(x) = Pr(X = x) = \begin{cases} 1-q, & x = 0 \\ q, & x = b \\ 0, & \text{elsewhere,} \end{cases}$$

  where q, the probability of a claim during the year and b, insurer pay amount if the insured dies with in a year of policy issue and nothing if insured survives the year.

- The distribution function is

$$F_X(x) = Pr(X \leq x) = \begin{cases} 0, & x < 0 \\ 1-q, & 0 \leq x \leq b \\ 1, & x \geq b \end{cases}$$

- Mean, $E[X] = bq$ and Variance, $Var(X) = b^2q(1-q)$.
- Conditional Expectations and Variance:

$$E[X] = E[E[X \mid I]] = \mu q,$$
$$Var[X] = Var(E[X \mid I]) + E[Var(X \mid]) = \mu^2 q(1-q) + \sigma^2 q.$$

- Distribution function of sum of two independent random variables is,

$$F_S(s) = \sum_{all\ y \leq s} F_X(s-y) f_Y(y),$$

  and the probability function is

$$f_S(s) = \sum_{all\ y \leq s} f_X(s-y) f_Y(y).$$

- MGF is $M_S(t) = E[e^{tS}]$.

Normal Approximation

- The approximate distribution of S is,

$$Pr(S \leq s) = Pr\left(\frac{S - E(S)}{\sqrt{Var(S)}} \leq \frac{s - E(S)}{\sqrt{Var(S)}}\right)$$

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Collective Risk Model

$$S = \sum_{j=1}^{N} X_j$$

$N, X_1, X_2, \ldots$ are independent random variables. Each X_j has d.f. $P(x)$, m.g.f. $M_X(t)$, and $p_k = E[X^k] \quad k = 1, 2, \ldots$

$$P^{*0}(x) = \begin{bmatrix} 1 & x \geq 0 \\ 0 & \text{elsewhere} \end{bmatrix}$$

$$P^{*n}(x) = \begin{cases} \sum_{j=0}^{x} p(x-j)P^{*(n-1)}(j), \text{ or} \\ \int_0^x p(x-y)P^{*(n-1)}(y)dy \end{cases}$$

Definitions	Distribution Function, $F_S(x)$	Restrictions on Parameters	Moment Generating Function, $M_S(t)$	Mean	Variance
General	$\sum_{n=0}^{\infty} \Pr(N=n)P^{*n}(x)$	—	$M_N[\log M_X(t)]$	$p_1 E[N]$	$E[N](p_2 - p_1^2) + p_1^2 \text{Var}(N)$
Compound Poisson	$\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} P^{*n}(x)$	$\lambda > 0$	$e^{\lambda[M_X(t)-1]}$	λp_1	λp_2
Compound Negative Binomial	$\sum_{n=0}^{\infty} \binom{r+n-1}{n} p^r q^n P^{*n}(x)$	$0 < p < 1$, $q = 1-p$, $r > 0$	$\left[\frac{p}{1-qM_X(t)}\right]^r \quad qM_X(t) < 1$	$\frac{rqp_1}{p}$	$\frac{rqp_2}{p} + \frac{rq^2p_1^2}{p^2}$
Compound Poisson Inverse Gaussian	no known closed form	$\alpha > 0$, $\beta > 0$	$\exp\left\{\alpha\left[1-\left\{1-\frac{2[M_X(t)-1]}{\beta}\right\}^{1/2}\right]\right\}$	$\frac{\alpha}{\beta} p_1$	$\frac{\alpha}{\beta}\left(p_2 + \frac{p_1^2}{\beta}\right)$

Ruin Theory

- Surplus process is $U(t) = u + ct - S(t), t \geq 0$ where $U(t)$, the insurer's random capital at time t; $u = U(0)$, the initial surplus; c, the constant premium income per unit of time; $S(t) = X_1 + X_2 + \cdots + X_{N(t)}$.
- Ruin probability, $\psi(u) = Pr(T < \infty)$ where

$$T = \begin{cases} min\{t \mid t \geq 0 \& U(t) < 0\}; \\ \infty, \; if \; U(t) \geq 0 \; \forall t. \end{cases}$$

- Adjustment coefficient (continuos case): $1 + (1+\theta)\mu R = M_X(R)$
- Loading factor, $c = (1+\theta)\lambda\mu$.