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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	TECHNIQUES OF OPTIMIZATION II
COURSE CODE	:	BWA 40703
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 Consider the nonlinear optimization problem

$$\text{Minimize} \qquad 2x_1^2 + x_2^2 + (x_1 + x_2)^2 - 20x_1 - 16x_2\,,$$

subject to

$$x_1 + x_2 \le 5, \qquad x_1 \ge 0, \qquad x_2 \ge 0.$$

(a) Expand the penalty function, given that

$$P(x) = \frac{1}{2}\sum_{i=1}^{3}(\max[0, g_i(x)])^2\,.$$

(3 marks)

(b) Define the penalty objective function.

(3 marks)

(c) Show that the first-order necessary conditions are

$$6x_1 + 2x_2 - 20 + c(\max[0, x_1 + x_2 - 5]) - c(\max[0, -x_1]) = 0,$$
$$2x_1 + 4x_2 - 16 + c(\max[0, x_1 + x_2 - 5]) - c(\max[0, -x_2]) = 0.$$

(6 marks)

(d) Deduce the solution

$$x_1 = \frac{7c^2 + 33c + 36}{3c^2 + 14c + 15} \text{ and } x_2 = \frac{8c + 14}{3c + 5}$$

as c approaches ∞.

(8 marks)

Q2 Consider a constrained optimization problem

$$\text{Minimize} \quad x_1^2 + 2x_2^2,$$

subject to

$$1 - x_1 - x_2 \leq 0.$$

The barrier function is defined by

$$B(x) = -\log(x_1 + x_2 - 1).$$

(a) Write an equivalent unconstrained problem.

(4 marks)

(b) Indicate that the first-order necessary conditions are given by

$$2x_1(x_1 + x_2 - 1) - \mu = 0,$$
$$4x_2(x_1 + x_2 - 1) - \mu = 0.$$

(6 marks)

(c) Prove that the solution for **Q2 (b)** is given by

$$x_1 = \frac{1 \pm \sqrt{1 + 3\mu}}{3} \text{ and } x_2 = \frac{1 \pm \sqrt{1 + 3\mu}}{6}.$$

(10 marks)

Q3 Consider a nonlinear optimization problem

$$\text{Minimize} \qquad 2x_1^2 + x_2^2 - 2x_1x_2 - 4x_1 - 6x_2,$$

subject to

$$\begin{aligned} x_1 + x_2 &\le 8, \\ -x_1 + 2x_2 &\le 10, \\ -x_1 &\le 0, \\ -x_2 &\le 0. \end{aligned}$$

(a) Obtain the coefficient matrix for the active constraints and the inactive constraints. The initial point is $x_1 = (0,\ 0)^{\mathrm{T}}$.

(4 marks)

(b) Calculate the projection matrix, that is,

$$P = I - A_1^{\mathrm{T}}(A_1 A_1^{\mathrm{T}})^{-1} A_1 .$$

(8 marks)

(c) Show that multiplier $u = (-4,\ -6)^{\mathrm{T}}$, given that

$$u = -(A_1 A_1^{\mathrm{T}})^{-1} A_1 \nabla f(x_1) .$$

(8 marks)

Q4 Assume that $\mathbf{x}^*$ is a regular point, then there will be a corresponding Lagrange multiplier vector $\boldsymbol{\lambda}^*$ such that

$$\nabla f(\mathbf{x}^*) + (\boldsymbol{\lambda}^*)^{\mathrm{T}} \nabla \mathbf{h}(\mathbf{x}^*) = \mathbf{0},$$

and the Hessian of the Lagrangian

$$\mathbf{L}(\mathbf{x}^*) = \mathbf{F}(\mathbf{x}^*) + (\boldsymbol{\lambda}^*)^{\mathrm{T}} \mathbf{H}(\mathbf{x}^*)$$

must be positive semidefinite on the tangent subspace

$$M = \{\mathbf{x} : \nabla \mathbf{h}(\mathbf{x}^*) \cdot \mathbf{x} = \mathbf{0}\}.$$

(a) Show that the dual function ϕ has the gradient

$$\nabla \phi(\boldsymbol{\lambda}) = \mathbf{h}(\mathbf{x}(\boldsymbol{\lambda}))^{\mathrm{T}}.$$

(9 marks)

(b) Determine that the Hessian of the dual function is

$$\Phi(\boldsymbol{\lambda}) = -\nabla \mathbf{h}(\mathbf{x}(\boldsymbol{\lambda}))\mathbf{L}^{-1}(\mathbf{x}(\boldsymbol{\lambda}), \boldsymbol{\lambda}) \nabla \mathbf{h}(\mathbf{x}(\boldsymbol{\lambda}))^{\mathrm{T}}.$$

(11 marks)

Q5 Assume that the management has decided to produce $P = 6{,}000$ units of a given product line consisting of three individual items. The allocation of the total quantity among the three items will be decided by the following mathematical model:

$$\text{Minimize} \qquad C = \sum_{i=1}^{3}\left(h_i \frac{Q_i}{2} + S_i \frac{d_i}{Q_i}\right),$$

subject to

$$\sum_{i=1}^{3} Q_i = P,$$

where

Q_i is the production quantity for item i (in units),

h_i is the inventory holding cost for item i (in RM per month × unit),

S_i is the setup cost for item i (in RM),

d_i is the demand for item i (in units per month),

P is the total amount to be produced (in units).

(a) Indicate the equivalent unconstrained minimization problem.

(4 marks)

(b) Derive the first-order necessary conditions.

(4 marks)

(c) Show that the optimal production quantity for item i is

$$Q_i^* = \sqrt{\frac{2S_i d_i}{h_i + 2\lambda}}.$$

(3 marks)

(d) Evaluate the production quantity for $i = 1, 2, 3$, where the values of the parameters are listed below

$$\lambda = 1,\ h_1 = 1,\ h_2 = 1,\ h_3 = 2,\ S_1 = 100,\ S_2 = 50,\ S_3 = 400,$$
$$d_1 = 20{,}000,\ d_2 = 40{,}000,\ d_3 = 40{,}000.$$

(9 marks)

– END OF QUESTIONS –