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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	CALCULUS
COURSE CODE	:	BWC 10303
PROGRAMME CODE	:	BWC
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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Q1 (a) Identify the limit of the following exists by testing from both sides.

(i) $\lim_{x\to 0}\frac{1}{x}$

(3 marks)

(ii) $\lim_{x\to 0}\frac{3}{x^2}$

(3 marks)

(iii) $\lim_{x\to -3}\frac{-2}{3+x}$

(3 marks)

(b) **Figure Q1(b)** represents the function of $f(x)=\frac{4x+10}{x^2-2x-15}$.

(i) Find the values of x where the discontinuity of the function occured.

(2 marks)

(ii) Identify with evidence of how the values of x can be found if **Figure Q1(b)** is not presented.

(4 marks)

(c) Suppose that the amount of water in a holding tank at t minutes is given by $V(t)=2t^2-16t+35$. Is the volume of water in the tank

(i) increasing or decreasing at $t = 1$ minute?

(4 marks)

(ii) increasing or decreasing at $t = 5$ minute?

(3 marks)

(iii) changing faster at $t = 1$ or $t = 5$ minutes?

(1 mark)

(iv) ever not changing? If so, when?

(2 marks)

Q2 (a) Differentiate each of the followings.

(i) $f(z) = \sin\left(ze^{z}\right)$

(3 marks)

(ii) $h(t) = \left(\dfrac{2t+3}{6-t^2}\right)^3$

(3 marks)

(iii) $y(x) = \tan\left(\sqrt[3]{3x^2} + \ln\left(5x^4\right)\right)$

(3 marks)

(b) A spot light is on the ground 20 feet away from a wall and a 6 feet tall person is walking towards the wall at a rate of 2.5 feet/sec. Demonstrate how fast is the height of the shadow changing when the person is 8 feet from the wall? Is the shadow increasing or decreasing in height at this time?

(6 marks)

(c) Illustrate the intervals for the following function where the function is increasing and decreasing and the intervals where the function is concave up and concave down. Use this information to sketch the graph.

$$p(z) = 3z^5 - 5z^3 + 3$$

(10 marks)

Q3 (a) Suppose that you want to construct a closed storage box with a square base and you only have 10 m^2 area of cardboard material. By assuming all the material is going to be used in the construction process, analyze the maximum volume that the box can have.

(10 marks)

(b) Integrate each of the followings.

(i) $\int \left(4 - s^2\right)\left(\sqrt[3]{s} + s\right) ds$

(2 marks)

(ii) $\int \dfrac{3 - 5\sin^2\phi}{\sin^2\phi} d\phi$

(2 marks)

(iii) $\int (8t-1)3e^{4t^2-t} dt$

(2 marks)

(c) Determine the function of $f(x)$ with known constant value, given that $f'(x) = -9 + 4x^3 + 7e^x + 2\sin x$ and $f(0) = 15$.

(4 marks)

(d) Solve the following definite integral using substitution rule.

$$\int_{\frac{1}{50}}^{2} \frac{e^{\frac{2}{t}}}{t^2}\, dt$$

(5 marks)

Q4 (a) Demonstrate the area of the region bounded by $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$ and $x = 5$ as shown in **Figure Q4(a)**.

(6 marks)

(b) Apply method of cylinder in finding the volume of the solid obtained by rotating about the x axis and the region bounded by $y = \sqrt[3]{x}$, $x = 8$ and x axis.

(9 marks)

(c) Use Simpson's $\frac{1}{3}$ rule to approximate $\int_{-2}^{0} \frac{1}{1+e^x}\, dx$ with $n = 8$.

(10 marks)

– **END OF QUESTIONS** –

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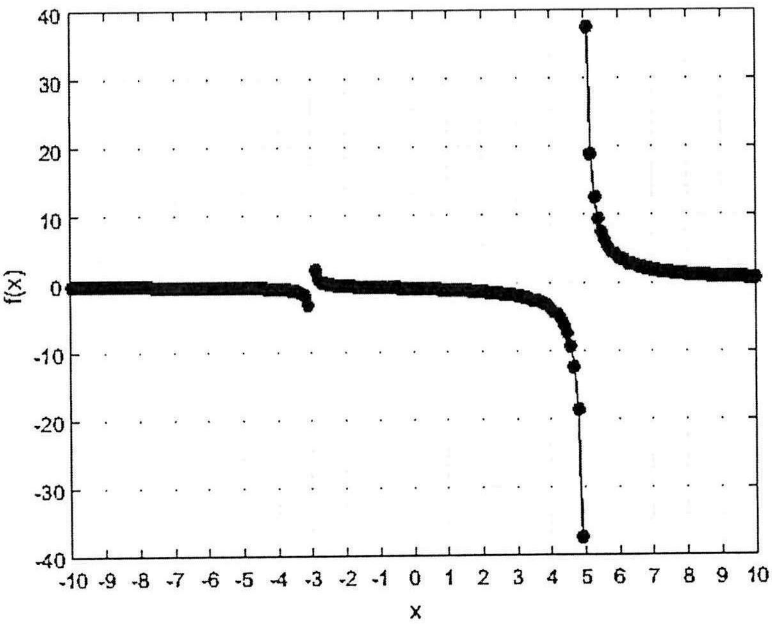

Figure Q1(b)

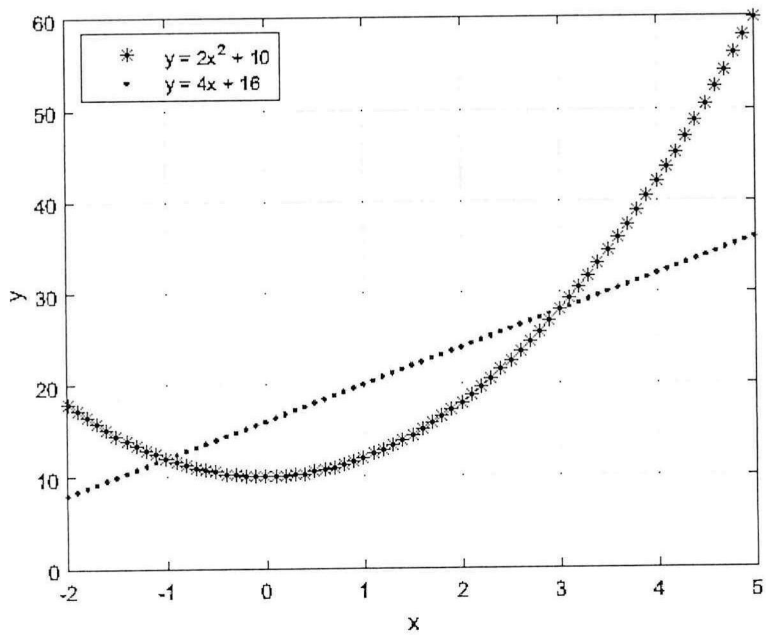

Figure Q4(a)

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Formulae

Indefinite Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x\,dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\,dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\,dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\,dx = \operatorname{sech}^{-1}|x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\,dx = \operatorname{csch}^{-1}|x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\,dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

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Formulae

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$
$$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$
$$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$$
$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

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CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left| \dfrac{d^2y}{dx^2} \right|}{\left[1 + \left(\dfrac{dy}{dx} \right)^2 \right]^{3/2}}$$

$$\kappa = \frac{| \dot{x}\ddot{y} - \dot{y}\ddot{x} |}{\left[\dot{x}^2 + \dot{y}^2 \right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2}\, dx$$

$$L = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy} \right)^2}\, dy$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2}\, dt$$

$$S = 2\pi \int_{y_1}^{y_2} g(y) \sqrt{1 + \left(\frac{d}{dy}[g(y)] \right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x) \sqrt{1 + \left(\frac{d}{dx}[f(x)] \right)^2}\, dx$$