Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	CALCULUS 1
COURSE CODE	:	BWA 10203
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) In the same graph, sketch the following properties for the function f.

(i) $f(0) = f(2) = 1$,

(ii) $\lim_{x \to 2^-} f(x) = +\infty$ and $\lim_{x \to 2^+} f(x) = 0$,

(iii) $\lim_{x \to -1^-} f(x) = -\infty$ and $\lim_{x \to -1^+} f(x) = +\infty$, and

(iv) $\lim_{x \to +\infty} f(x) = 2$ and $\lim_{x \to -\infty} f(x) = +\infty$.

(5 marks)

(b) Evaluate the following limits.

(i) $\lim_{x \to 0} \dfrac{e^x - e^{2x}}{\sqrt{5x+1} - 1}$.

(4 marks)

(ii) $\lim_{x \to 0} \left(1 + \sin^{-1}(2x)\right)^{1/x}$.

(6 marks)

(c) Determine whether or not the following function is continuous at $x = -2$.

$$f(x) = \begin{cases} \dfrac{x+2}{x^3 + 2x^2 + x + 2}, & x < -2, \\ \dfrac{1}{x} - \dfrac{7}{5x}, & x \geq -2. \end{cases}$$

(5 marks)

Q2 (a) Find the derivative of the function $f(x) = \sqrt{5x - 2}$ using the first principle rule.

(4 marks)

(b) Differentiate each of the following functions.

(i) $f(x) = \left(\tan^{-1}(3x) + 2^x\right)^2$.

(3 marks)

(ii) $g(x) = (\sin x)^{\sqrt[3]{x}}$.

(6 marks)

(c) Find the values of a and b for the curve $x^2 y + ay^2 = b$ if the function is passing through point $(1, 1)$ and the tangent line at that particular point is $4x + 3y = 7$.

(7 marks)

Q3 (a) Find the linearization of $\tan x$ at $x = -\pi/4$ and then sketch both the curve and the linearization.

(5 marks)

(b) If two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an $R-$ohm resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 is decreasing at the rate of 1 ohm/sec and R_2 is increasing at the rate of 0.5 ohm/sec, at what rate is R changing when $R_1 = 75$ ohms and $R_2 = 50$ ohms?

(5 marks)

(c) Given $f(x) = x^4 e^{-x}$.

(i) Find all the critical number of $f(x)$.

(2 marks)

(ii) Find the extremum points and the interval which $f(x)$ is increasing and decreasing.

(4 marks)

(iii) Find the interval which $f(x)$ is concave upward and downward, and the inflection point, if any.

(4 marks)

Q4 (a) Find the general form of a function whose second derivative is $\sqrt{x}$.

(4 marks)

(b) Evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{x(1+(\ln x)^2)}\, dx$.

(5 marks)

(ii) $\displaystyle\int (\sin 3x - \cos x)^2\, dx$.

(6 marks)

(c) Evaluate $\displaystyle\int_0^{\pi/4} \frac{2}{\cos^2 x}\, dx$ using $t = \tan x$ substitution.

(5 marks)

Q5 (a) Evaluate $\int \frac{dx}{x\sqrt{4+x^{16}}}$.

(6 marks)

(b) Find the surface area generated when $y=\sqrt[3]{3x}$ from $y=-1$ to $y=0$ is rotated $360°$ about the y-axis.

(9 marks)

(c) If $y=\sqrt{1-x^2}\sin^{-1}x$, prove that

$$\left(1-x^2\right)\frac{dy}{dx}+xy=1-x^2 .$$

(5 marks)

- END OF QUESTIONS -

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FORMULA

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \text{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \text{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \text{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x + y) + \cos(x - y)$	

Logarithm	Inverse Hiperbolic
$a^x = e^{x \ln a}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$, any x.
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$, $x \geq 1$
	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

Definition of differentiation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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Indefinite Integrals	Integration of Inverse Functions
$\int k\,dx = kx + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \frac{dx}{\lvert a\rvert\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$
$\int \sin x\,dx = -\cos x + C$	$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0.$
$\int \cos x\,dx = \sin x + C$	$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a$
$\int \sec^2 x\,dx = \tan x + C$	$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right) + C,$ $0 < x < a.$
$\int \text{cosec}^2 x\,dx = -\cot x + C$	$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a}\text{cosech}^{-1}\left(\frac{x}{a}\right) + C,$ $0 < x < a.$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + C$	
$\int e^x\,dx = e^x + C$	
$\int \sinh x\,dx = \cosh x + C$	
$\int \cosh x\,dx = \sinh x + C$	
$\int \text{sech}^2 x\,dx = \tanh x + C$	
$\int \text{cosech}^2 x\,dx = -\coth x + C$	
$\int \text{sech}\,x \tanh x\,dx = -\text{sech}\,x + C$	
$\int \text{cosech}\,x \coth x\,dx = -\text{cosech}\,x + C$	