UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	CALCULUS II
COURSE CODE	:	BWA 10503
PROGRAMME CODE	:	BWA / BWQ
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Show that the Maclaurin series for $f(x) = \sin x$ is

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Then,

(i) evaluate $\int_0^1 \sin x^3 dx$.

(ii) find the first six terms of a series for $\cos x$ and $2\cos x \sin x$.

(12 marks)

(b) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$$

is absolutely convergent, conditionally convergent or divergent.

(8 marks)

Q2 (a) Given $f(x, y) = xy + \frac{e^x}{x^2 + 1}$. Verify that $f_{xy} = f_{yx}$.

(4 marks)

(b) The base radius r cm of a right-circular cone increases at 2 cm s^{-1} and its height h cm at 3 cm s^{-1}. Find the rate of increase in its volume when $r = 5$ cm and $h = 15$ cm, given that the volume $V = \frac{1}{3}\pi r^2 h$.

(6 marks)

(c) Find and classify all the extreme points (if exist) for $f(x, y) = 3x^2 y - 2xy + 5y^2$.

(10 marks)

Q3 (a) (i) Show that the limit of

$$\frac{xy^2}{x^2 + y^4}$$

does not exist when $(x, y) \to (0,0)$ by taking the limit along a straight line $y = mx$ and the parabola $x = y^2$.

(ii) Determine whether the function

$$f(x, y) = \begin{cases} \dfrac{x^4 - 4y^4}{x^2 + 2y^2}, & \text{if } (x, y) \neq (0,0) \\ 0, & \text{if } (x, y) = (0,0) \end{cases}$$

continuous at (0, 0) or not.

(9 marks)

(b) Evaluate

$$\iint_S \left(1 - x^2 - y^2\right) dS,$$

where S is the surface of the hemisphere $x^2 + y^2 + z^2 = 1$ that lies over the xy – plane.

(11 marks)

Q4 (a) Evaluate $\int_{y=0}^{y=2} \int_{x=0}^{x=y^2} y^3 \, dx \, dy$ by interchange the order of integration.

(10 marks)

(b) Find the centroid of the circular lamina which is above the x-axis and between the circle $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$.

(10 marks)

Q5 (a) The sphere of radius a centred at the origin is expressed in rectangular coordinates as $x^2 + y^2 + z^2 = a^2$, and hence its equation in cylindrical coordinates is $r^2 + z^2 = a^2$. Use this equation and a polar double integral to find the volume of the sphere.

(10 marks)

(b) Evaluate

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} dz \, dy \, dx,$$

by changing it to spherical coordinate.

(10 marks)

- END OF QUESTIONS -

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Formulae

Area of region = $\iint_R dA$, $dA = dxdy$ or $dA = dydx$

Volume = $\iint_R f(x,y)dA$, $dA = dxdy$ or $dA = dydx$

Volume = $\iiint_G dV$, $dV = dzdydx$ or $dzdxdy$ or $dydzdx$ or $dydxdz$ or $dxdydz$ or $dxdzdy$

Volume = $\iiint_G dV = \iiint_G dzrdrd\theta$, $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $x^2 + y^2 = r^2$, $0 \le \theta \le 2\pi$.

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$,
$0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$ and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Surface Area = $\iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$, $dA = dxdy$ or $dA = dydx$ or $dA = rdrd\theta$

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

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Lamina

Mass, $m = \iint_R \delta(x,y)\,dA$

The moment of mass: $M_y = \iint_R x\delta(x,y)\,dA$, $M_x = \iint_R y\delta(x,y)\,dA$

Center of mass, $(\bar{x},\bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m}\right)$

Centroid for homogeneous lamina: $\bar{x} = \dfrac{1}{\text{area}}\iint_R x\,dA$, $\bar{y} = \dfrac{1}{\text{area}}\iint_R y\,dA$

Moment Inersia : $I_y = \iint_R x^2\delta(x,y)\,dA$,

$$I_x = \iint_R y^2\delta(x,y)\,dA\ ,$$

$$I_z = I_0 = \iint_R (x^2+y^2)\delta(x,y)\,dA = I_x + I_y$$

Solid

Mass, density × volume

Moment about the plane: $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$,

$$M_{xz} = \iiint_G y\delta(x,y,z)\,dV,$$

$$M_{xy} = \iiint_G z\delta(x,y,z)\,dV$$

Center of gravity, $(\bar{x},\bar{y},\bar{z})$

$$\bar{x} = \frac{1}{m}\iiint_G x\delta(x,y,z)\,dV = \frac{M_{yz}}{m},\quad \bar{y} = \frac{1}{m}\iiint_G y\delta(x,y,z)\,dV = \frac{M_{xz}}{m},\quad \bar{z} = \frac{1}{m}\iiint_G z\delta(x,y,z)\,dV = \frac{M_{xy}}{m}$$

Centroid : $\bar{x} = \dfrac{1}{V}\iiint_G x\,dV$, $\bar{y} = \dfrac{1}{V}\iiint_G y\,dV$, $\bar{z} = \dfrac{1}{V}\iiint_G z\,dV$

Moment Inersia : $I_x = \iiint_G (y^2+x^2)\delta(x,y,z)\,dV$,

$$I_y = \iiint_G (x^2+z^2)\delta(x,y,z)\,dV$$

$$I_z = \iiint_G (x^2+y^2)\delta(x,y,z)\,dV$$