Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	LINEAR ALGEBRA
COURSE CODE	:	BWA 10303
PROGRAMME CODE	:	BWA / BWQ
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

Q1 (a) Consider a matrix

$$A=\begin{bmatrix} -1 & 7 & 3 & 1 & -4 \\ 7 & 0 & 2 & 9 & 3 \\ 3 & 2 & 3 & 2 & 7 \end{bmatrix}.$$

State the size of the matrix and find $a_{12}+(a_{23}-a_{24})^2-\sqrt{a_{35}}$.

(3 marks)

(b) Given $A=\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$.

(i) Calculate A^2 and A^3.

(ii) Verify that $A^3-2A^2-7A+14I=0$.

(iii) Hence, express A^4 in term of A^2, A and I.

(9 marks)

(c) Show that $A=\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$ is an idempotent matrix.

(2 marks)

Q2 Let $A=\begin{bmatrix} 6 & -3 & 0 \\ 1 & 4 & -3 \\ 5 & -5 & 7 \end{bmatrix}$ is an invertible matrix.

(a) Find the minor, M_{ij} and cofactor, C_{ij} of element a_{23} and a_{33}.

(3 marks)

(b) Then, by using results in **Q2(a)**, compute $|A|$ by cofactor expansion.

(2 marks)

(c) Determine A^{-1} using adjoint of A.

(6 marks)

Q3 Consider a system of linear equations

$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 0 \\ 5x_2 + 9x_3 &= -4 \\ 3x_2 - x_3 &= 7 \end{aligned}$$

(a) Solve the linear system using Cramer's rule.

(7 marks)

(b) List all properties of reduced row echelon form (RREF) matrix.

(4 marks)

(c) Then, by employing RREF, solve the linear system using Gauss-Jordan elimination method.

(7 marks)

Q4 (a) Consider the vectors $u_1 = (1,\ 2)$ and $u_2 = (3,\ 4)$ in R^2. Write $u_3 = (11,\ 16)$ as a linear combination of u and v.

(4 marks)

(b) Given the vectors $v_1 = (1,\ 1,\ 2)$, $v_2 = (1,\ 2,\ 5)$ and $v_3 = (5,\ 3,\ 4)$.

(i) Determine whether the vectors are linearly dependent or not.

(ii) Hence, is the vectors form a basis of R^3?

(8 marks)

(c) Examine the rank of $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$.

(6 marks)

Q5 Let $u_1 = (1,\ 3,\ -4,\ 2)$, $u_2 = (4,\ -2,\ 2,\ 1)$ and $u_3 = (5,\ -1,\ -2,\ 6)$ in R^4.

(a) Show that $\langle 3u_1 - 2u_2,\ u_3 \rangle = 3\langle u_1,\ u_3 \rangle - 2\langle u_2,\ u_3 \rangle$.

(6 marks)

(b) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace U of R^4 spanned by u_1, u_2 and u_3.

(9 marks)

Q6 (a) State the relationship between the eigenvalues and the invertibility of a matrix.

(2 marks)

(b) Let $A = \begin{bmatrix} 3 & 4 \\ 8 & -1 \end{bmatrix}$.

(i) Determine the characteristic polynomial and characteristic equation of A.

(ii) Hence, determine all eigenvalues and corresponding eigenvectors for A.

(iii) Is A invertible? Explain your reason.

(18 marks)

(c) Use the Cayley-Hamiton theorem to show that matrix A in **Q6(b)** is satisfies its own characteristic equation.

(4 marks)

- END OF QUESTIONS -