Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	LINEAR PROGRAMMING
COURSE CODE	:	BWA 20403
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019/ JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 A community council must decide which recreation facilities to construct in its community. Four new recreation facilities have been proposed which are swimming pool, tennis center, athletic field and gymnasium. The council wants to construct facilities that will maximize the expected daily usage by the residents of the community, subject to land and cost limitations. The expected daily usage, cost and land requirements for each facility are as in **Table Q1**.

Table Q1

Recreation Facilities	Expected Usage (People/ day)	Cost	Land Requirement (Acres)
Swimming Pool	300	$35,000	4
Tennis Center	90	$10,000	2
Athletic Field	400	$25,000	7
Gymnasium	150	$90,000	3

The community has $120,000 construction budget and 12 acres of land. The swimming pool and tennis center must be built on the same part of the land parcel. However, only one of these two facilities can be constructed. The council wants to identify which of the recreation facilities to construct to maximize the expected daily usage. Formulate the integer linear programming model of the problem, but do not solve.

(6 marks)

Q2 Consider the following linear programming model

Maximize $z = 10x + 30y$

Subject to

$$\begin{aligned} -2x + 3y &\leq 6 \\ 3x - 12y &\leq 12 \\ 2x + 3y &\leq 20 \\ 2y &\leq 7 \\ x, y &\geq 0. \end{aligned}$$

(a) Solve the above linear programming problem graphically.

(7 marks)

(b) Describe on the optimal solution if the objective function is changed to $z = 15x + 30y$.

(3 marks)

(c) Describe on the optimal solution if the third main constraint is changed to $2y \leq 9$.

(6 marks)

Q3 Solve the following linear programming problem using the Two – Phase method.

$$\begin{aligned} &\text{Minimize } z = 2x_1 - x_2 + x_3 \\ &\text{Subject to} \\ &\quad 2x_1 + x_2 + x_3 \le 5 \\ &\quad x_1 - x_2 + 2x_3 = 4 \\ &\quad x_1 + 3x_2 + x_3 \ge 2 \\ &\quad x_1, x_2, x_3 \ge 0 \end{aligned}$$

(18 marks)

Q4 Consider the following linear programming model

$$\begin{aligned} &\text{Maximize } z = 2x_1 + 5x_2 + x_3 \\ &\text{Subject to} \\ &\quad x_1 + 3x_2 + 4x_3 \le 60 \\ &\quad 3x_1 + x_2 + 2x_3 \ge 45 \\ &\quad x_1, x_2, x_3 \ge 0. \end{aligned}$$

(a) Write the dual of the above primal problem.

(6 marks)

(b) Find the optimal solution of the above linear programming model by the generalized simplex method.

(16 marks)

Q5 Describe how the following situations are recognized in an iteration of simplex method. Give an example for each situation.

(a) Existence of an alternative optimal solution.

(3 marks)

(b) Existence of an unbounded solution.

(3 marks)

Q6 Consider the following integer linear programming model.

$$\begin{aligned} &\text{Maximize } z = 6x_1 - 5x_2 \\ &\text{Subject to} \\ &\quad 2x_1 + 5x_2 \le 19 \\ &\quad 2x_1 - 5x_2 \le 8 \\ &\quad x_1, x_2 \ge 0 \text{ and integers.} \end{aligned}$$

Find the optimal integer solution using the branch-and-bound (B&B) method.

(16 marks)

Q7 The optimal simplex tableau for the following linear programming model

$$\begin{aligned}
&\text{Maximize } z = 4x_1 + 5x_2 - x_3 \\
&\text{Subject to} \\
&\qquad 2x_1 + x_2 + 4x_3 \le 120 \\
&\qquad 2x_1 + 3x_2 - x_3 \le 200 \\
&\qquad x_1, x_2, x_3 \ge 0
\end{aligned}$$

is given in **Table Q7**.

Table Q7: Optimal tableau

Basic	x_1	x_2	x_3	s_1	s_2	Solution
z	0	0	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	360
x_1	1	0	$\frac{13}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	40
x_2	0	1	$-\frac{5}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	40

Conduct sensitivity analysis by independently investigating each of the following changes in the original model. Check whether each of the changes will affect the current solution. If so, find the new solution.

(a) The objective function is changed to $z = 4x_1 + 5x_2 - 2x_3$.

(6 marks)

(b) The right-hand side value of the second constraint is changed to 150.

(4 marks)

(c) A new variable x_4 is added with coefficients in the objective function $c_4 = 7$, the first constraint $a_{14} = 2$ and the second constraint $a_{24} = -1$.

(6 marks)

- END OF QUESTIONS -