UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	MATHEMATICAL PHYSICS
COURSE CODE	:	BWC 20103
PROGRAMME CODE	:	BWC
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

Q1 (a) Given a function, $f(x,y)=\sqrt{9-x^2-y^2}$. Find the domain and range of $f(x,y)$ and sketch the graph of the function.

(6 marks)

(b) Find the local extreme value of the function $f(x,y)=1-x^2+y^2$.

(7 marks)

(c) Calculate the area of region enclosed by the cardiod $r=1+\cos\theta$.

(7 marks)

Q2 (a) Calculate the volume of the solid that lies between a paraboloid $z=x^2+y^2$ and the upper hemisphere $x^2+y^2+z^2=2$ by using the cylindrical coordinate.

(10 marks)

(b) Using $x=\rho\sin\phi\cos\theta$, $y=\rho\sin\phi\sin\theta$, $z=\rho\cos\phi$, $x^2+y^2+z^2=\rho^2$ and $dV=\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$, convert the rectangular coordinate integral into spherical coordinate integral to solve

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z^2\sqrt{x^2+y^2+z^2}\, dzdydx$$

(10 marks)

Q3 (a) It takes three different ingredients in g/cm³ A, B, and C, to produce a certain chemical substance. A, B, and C have to be dissolved in water separately before they react to form the chemical substance. Suppose that solution A at 1.5 g/cm³ mixed with solution B at 3.6 g/cm³ and solution C at 5.3 g/cm³ produced 25.07 g of the amount chemical substance. If the amount of A, B, C in these solutions are changed to 2.5 g/cm³, 4.3 g/cm³ and 2.4 g/cm³, respectively, then 22.36 g of the chemical substance is produced. Finally, if the amount are 2.7 g/cm³, 5.5 g/cm³ and 3.2 g/cm³, respectively, then 28.14 g of the chemical substance is produced. Let x, y, z be the corresponding volumes (in cubic centimetres) of the solutions containing A, B, and C. Based on the problem,

(i) construct a system of linear equations.

(4 marks)

(ii) using Gauss Seidel iteration method, determine the volumes (in cubic centimetres) of the chemical substance containing A, B, and C.

(8 marks)

(b) Use power method to approximate a dominant eigenvalue and the corresponding eigenvector of a matrix $\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$ correct to 3-significant figures for 5 iterations with the initial eigenvector $[1 \quad 1 \quad 1]^T$.

(8 marks)

Q4 (a) What is a complex number? Compute $\sqrt[3]{(-15-8i}$. Show the graphical representation of the result.

(8 marks)

(b) The voltage in a circuit is $30+10\hat{j}$ V. The impedance is $5+3\hat{j}\,\Omega$. Calculate the current. Why do you use $\hat{j}$ instead of $\hat{i}$?

(4 marks)

(c) Given that $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

Prove that $\cos^4\theta = \dfrac{1}{8}\cos 4\theta + \dfrac{1}{2}\cos 2\theta + \dfrac{3}{8}$

(8 marks)

Q5 (a) A periodic function $f(x)$ is defined as

$$f(x) = 4 - x^2, \quad \text{for } -2 < x < 2$$

and

$$f(x) = f(x+4).$$

(i) Sketch the graph of the above function over $-6 < x < 6$.

(3 marks)

(ii) Determine the Fourier series of the above function.

(5 marks)

(b) A rod of length π is fully insulated along its sides. Its temperature at x is initially $x(\pi - x)$. At $t = 0$ the ends are dipped into ice and held at temperature of 0°C. The heat equation is given by $\dfrac{\partial u}{\partial t} = 2\dfrac{\partial^2 u}{\partial x^2}$, where $0 < x < \pi$ and $t > 0$. Deduce the first three nonzero terms of the particular solution $u(x,t)$ if $u(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\dfrac{n\pi x}{l}\right) e^{-\frac{n^2\pi^2c^2t}{l^2}}$.

(12 marks)

– END OF QUESTIONS –