Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BWM 12303
PROGRAMME CODE	:	BNA/ BNB/ BND/ BNF/ BNG/ BNH/ BNL/ BNM
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

TERBUKA

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Show that the given equation is homogeneous and find the solution.

$$\left(y+\sqrt{x^2+y^2}\right)dx - x\,dy = 0 \quad \text{with} \quad y(1)=0.$$

(12 marks)

(b) Show that the given equation is exact and find the solution.

$$\left(3x^2y+\frac{y}{x}\right)dx+\left(x^3+\ln x\right)dy=0.$$

(13 marks)

Q2 (a) Find the solution of the homogeneous initial value problem.

$$y''+4y'+5y=0, \quad y(0)=1 \quad \text{and} \quad y'(0)=2.$$

(10 marks)

(b) By using method of variation of parameters, find the solution of the differential equation.

$$y''-2y'-3y=64xe^{-x}.$$

(15 marks)

Q3 (a) The Laplace transform for an unknown function $g(t)$ is given as

$$L\{g(t)\}=\frac{4}{s^2-4s+3}.$$

Find the inverse Laplace transform by using **convolution theorem** in order to determine the function $g(t)$.

(8 marks)

(b) (i) Express $\dfrac{1}{(s+1)(s+2)^2}$ in partial fraction form.

(5 marks)

(ii) Find the inverse Laplace for **Q3** (b)(i).

(5 marks)

(iii) Use the result in **Q3** (b)(ii) to solve the following initial-value problem

$$\frac{dy}{dt}+y=te^{-2t}, \quad y(0)=0.$$

(7 marks)

Q4 (a) By using second order Taylor series method, solve the following initial-value problem (IVP) below:

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1, \quad x = 0(0.05)0.2.$$

(10 marks)

(b) Given the boundary-value problem (BVP) as below:

$$xy'' - 3xy' + 2y = 0,$$

in the interval $[1, 2]$,with the boundary conditions, $y(1) = 4$ and $y(2) = 14$.

(i) Derive a system of linear equations (in matrix-vector form) using the finite difference method by taking $\Delta x = h = 0.25$.

(12 marks)

(ii) Hence, solve the system.

(3 marks)

- END OF QUESTIONS –

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW}\,dx + A$, $v = \int \frac{y_1 f(x)}{aW}\,dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Taylor method

$$y_{i+1} = y_i + hy_i' + \frac{h^2}{2!}y_i'' + ... + \frac{h^n}{n!}y_i^{(n)}$$

Finite difference method

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h},$$

$$y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}.$$

Thomas Decomposition method

$$\begin{pmatrix} d_1 & e_1 & 0 & \cdots & 0 \\ c_2 & d_2 & e_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & c_{n-1} & d_{n-1} & e_{n-1} \\ 0 & \cdots & 0 & c_n & d_n \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_1 & 0 & 0 & \cdots & 0 \\ c_2 & \alpha_2 & 0 & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & c_{n-1} & \alpha_{n-1} & 0 \\ 0 & \cdots & 0 & c_n & \alpha_n \end{pmatrix} \begin{pmatrix} 1 & \beta_1 & 0 & \cdots & 0 \\ 0 & 1 & \beta_2 & & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & 0 & 1 & \beta_{n-1} \\ 0 & \cdots & 0 & 0 & 1 \end{pmatrix}$$

where $\alpha_1 = d_1$

$$\alpha_i = d_i - c_i\beta_{i-1}, \quad i = 2,3,4,...,n$$

$$\beta_i = \frac{e_i}{\alpha_i}, \quad i = 1,2,3,...,n-1$$

Solve $Ly = b$ using forward substitution algorithm.

$$y_1 = \frac{b_1}{\alpha_1}, \quad y_i = \frac{(b_i - c_i y_{i-1})}{\alpha_i}, \quad i = 2,3,4,...,n$$

Solve $Ux = y$ using backward substitution algorithm.

$$x_n = y_n, \quad x_i = y_i - \beta_i x_{i+1}, \quad i = n-1, n-2, ..., 1$$