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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY III
COURSE CODE	:	BWM 22403
PROGRAMME CODE	:	BNG / BNL / BNM / BNT
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) ALL CALCULATIONS MUST BE IN **THREE (3) DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Determine whether the function

$$f(x,y)=\begin{cases}\dfrac{x^4+y^4}{\sqrt{x^4+y^4+1}-1}, & (x,y)\neq(0,0)\\[2ex] 2\quad, & (x,y)=(0,0)\end{cases}$$

is continuous at (0, 0) or not.

(6 marks)

(b) Find $\dfrac{dz}{dt}$ using the chain rule if $z=e^x-e^y,\ x=\ln t^2,\ y=t^3$.

(7 marks)

(c) Given $\int_0^1\int_x^1 \sin(\pi y^2)\,dy\,dx$. Evaluate the double integrals by changing the order of integration.

(8 marks)

(d) Calculate the volume of the solid between the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $y+z=4$ using the cylindrical coordinates.

(9 marks)

Q2 (a) Given nonlinear equation $f(x)=\sin x+\cos\left(1+x^2\right)-1$. Estimate the root of $f(x)$ in the interval [1,2] using secant method. Iterate until $|f(x_i)|<0.005$.

(7 marks)

(b) By the Kirchoff's law, the currents $I_1,\ I_2$ and I_3 in a three loop current network with five resistors R_1, R_2, R_3, R_4, R_5 and two voltage sources V_1 and V_2 can be described by the following system of linear equations:

$$\begin{aligned}-V_1 + R_1I_1 + R_2\left(I_1-I_2\right) &= 0\\ R_2\left(I_2-I_1\right) + R_3I_2 + R_4\left(I_2-I_3\right) &= 0\\ R_4\left(I_3-I_2\right) + R_5I_3 - V_2 &= 0\end{aligned}$$

where $R_1=R_2=R_3=R_4=R_5=1$ ohm and $V_1=5$ volts and $V_2=-6$ volts.

(i) Construct the problem in a matrix form.

(ii) Determine the currents $I_1,\ I_2$ and I_3 by solving the matrix in **Q2(b)(i)** using Thomas algorithm and Gauss-Seidel iteration method. Please use initial guess $\begin{bmatrix}3 & 2 & 4\end{bmatrix}^T$ for Gauss-Seidel iteration method.

(18 marks)

Q3 (a) Consider the data given in **Table Q3(a)**.

Table Q3(a)

x	0.3	0.35	0.4	0.45	0.5
$f(x)$	1.567	1.735	1.864	1.951	1.995

Find the approximate values of $f'(0.4)$ and $f''(0.4)$ with $h = 0.05$ and $h = 0.1$ using three point-central difference formula.

(6 marks)

(b) The moment of mass about y - axis of a thin plate with constant density, δ bounded by two curves $f(x)$ and $g(x)$ on the interval $[a,b]$ is given by

$$M_y = \delta\int_a^b x\left[f(x) - g(x)\right]dx.$$

Assume the constant density, δ is 2. Determine M_y for the plate bounded by $f(x) = 2\sin(2x)$ and $g(x) = 0$ on the interval $[0,\pi]$ using

(i) $\frac{1}{3}$ Simpson's rule with subinterval $n = 8$.

(ii) 2-point Gauss quadrature.

(19 marks)

Q4 (a) Given the data in **Table Q4(a)**.

Table Q4(a)

x	5	6	9	11	13
y	12	13	14	16	26

(i) Construct the polynomial of degree four that interpolates the data using Newton's divided-difference method.

(ii) Hence, estimate the value of y corresponding to $x = 7$.

(11 marks)

(b) Given a matrix

$$A = \begin{bmatrix} 10 & 7 & 3 \\ 7 & 8.5 & 2 \\ 3 & 2 & 1.5 \end{bmatrix}.$$

Determine the smallest eigenvalue, $\lambda_{\text{Smallest}}$ of A using inverse power method. Use initial eigenvector, $v^{(0)} = \begin{bmatrix} 0 & 1 & 0.5 \end{bmatrix}^T$. Do the iteration until $|m_{k+1} - m_k| < 0.005$.

(9 marks)

- END OF QUESTIONS -

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Formulas

Multiple integrals

Cylindrical coordinate : $x = r\cos\theta,\ y = r\sin\theta,\ z = z,\ x^2 + y^2 = r^2,\quad 0 \le \theta \le 2\pi.$

$$\iiint\limits_G f(x,y,z)\,dV = \iiint\limits_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

Nonlinear equations

Secant method : $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}, i = 0,1,2,\ldots$

System of linear equations

Thomas algorithm:

i	**1**	**2**	**…**	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Gauss-Seidel iteration method:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$$

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Interpolation

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x-x_0) + f_0^{[2]}(x-x_0)(x-x_1) + \ldots + f_0^{[n]}(x-x_0)(x-x_1)\ldots(x-x_{n-1})$$

Numerical differentiation

First derivatives

3-point central difference : $f'(x) \approx \dfrac{f(x+h)-f(x-h)}{2h}$

Second derivatives

3-point central difference : $f''(x) \approx \dfrac{f(x+h)-2f(x)+f(x-h)}{h^2}$

Numerical integration

$\frac{1}{3}$ Simpson's rule: $\displaystyle\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\text{ even}}}^{n-2} f_i\right]$

Gauss quadrature

For $\displaystyle\int_a^b f(x)dx$, $x = \dfrac{(b-a)t+(b+a)}{2}$

2-points: $\displaystyle\int_{-1}^{1} f(x)dx \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$

Eigenvalue

Power Method : $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \ k = 0,1,2,\ldots,$

Inverse Power Method : $\lambda_{\text{smallest}} = \dfrac{1}{\lambda_{\text{inverse}}}$