UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	NUMERICAL METHOD I
COURSE CODE	:	BWA 21303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) ALL CALCULATIONS MUST BE IN **THREE (3) DECIMAL** PLACES

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Absolute and relative error are two types of error which every experimental scientist should be familiar. The difference between the two errors is very significant. Discuss the difference, by not only stating the formulas.

(3 marks)

(b) Write down the Newton – Raphson method to approximate the cube root of a positive number c. Hence, calculate $\sqrt[3]{122}$ with $x_0 = 4$.

(7 marks)

(c) Given a system of linear equations as below:

$$\begin{aligned} 10a + b + 4c + 3d &= 13 \\ 4a + 5b + 14c + 3d &= 34 \\ 2a + 11b + 2c + 2d &= 15 \\ 2a + b + c + 9d &= 28 \end{aligned}$$

(i) Write down the equations in matrix form $Ax = B$.

(1 mark)

(ii) Solve the system using Gauss – Seidel iteration method.

(9 marks)

(iii) Given that the inverse of A is

$$A^{-1} = \frac{1}{591}\begin{pmatrix} \frac{139}{2} & -\frac{115}{6} & 4 & -\frac{53}{3} \\ -\frac{23}{3} & -\frac{49}{9} & \frac{173}{3} & -\frac{76}{9} \\ -\frac{43}{3} & \frac{448}{9} & -\frac{62}{3} & -\frac{65}{9} \\ -13 & -\frac{2}{3} & -5 & \frac{214}{3} \end{pmatrix}.$$

Determine and analyze the condition number of A.

(4 marks)

(iv) What can you conclude from the result in **Q1**(c)(iii)?

(1 mark)

Q2 (a) Consider the data points (0, 1), (1, 3), (1.5, 0) and (2, α). Use Lagrange polynomial interpolation to find the value of α if the coefficient of x^3 in the polynomial is 5.

(9 marks)

(b) By expanding $f(x+h)$ in a Taylor series up to three terms, deduce an expression for the truncation error e^T in the first derivative 2-point forward difference formula,

$$f'(x) = \frac{f(x+h) - f(x)}{h} + e^T .$$

(4 marks)

(c) A racehorse is running a 1 km race. A timer is set to record the distance, d (in meters) of the racehorse for every 10 seconds, t as shown in the **Table Q2(c)** below.

Table Q2(c)

t	0	10	20	30	40	50	60	70	80	90	100
d	0	45	104	196	310	408	495	620	721	816	902

(i) Approximate the speed of the horse after a minute of the race.
(4 marks)

(ii) During which part of the race is the horse running the fastest?
(2 marks)

(iii) Calculate the acceleration of the horse at its fastest part.
(4 marks)

(iv) Estimate how long does it takes to finish the race.
(2 marks)

Q3 (a) Consider a matrix

$$A = \begin{pmatrix} 4 & -1 & 1 \\ 1 & 1 & 1 \\ -2 & 0 & -6 \end{pmatrix}.$$

(i) Determine the interval of which the eigenvalues of matrix A above are contained by using Gerschgorin's theorem.
(7 marks)

(ii) Find the dominant eigenvalue and corresponding eigenvector for matrix A by using power method with $v^{(0)} = (1 \quad 0 \quad 0)^T$. Do your calculation until 6 iterations.
(5 marks)

(iii) Hence, find the smallest eigenvalue (in absolute value) and corresponding eigenvector for matrix A by using shifted power method with $v^{(0)} = (1 \quad 0 \quad 0)^T$ and $\varepsilon = 0.005$.
(6 marks)

(b) The error function,

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$

is important in probability and the theories of heat flow and signal transmission, which must be evaluated numerically because there is no elementary expression for the antiderivative of e^{-t^2}. Use the 3 – points Gauss quadrature to estimate $erf(1)$.
(7 marks)

Q4 (a) Euler's method is not an efficient numerical method on solving ordinary differential equations, but many of the ideas involved in numerical solution of differential equation are introduced with it. Discuss the advantages and disadvantages of Euler's method.

(4 marks)

(b) A physical phenomena is governed by a simple differential equation,

$$\frac{dv}{dt} = -\alpha(t)v + \beta(t) \text{ where } \alpha(t) = \frac{3t}{(1+t)} \text{ and } \beta(t) = 2(1+t)^3 e^{-t}.$$

Assume an initial value $v(0) = 1.0$. Solve the above first order initial value problem (IVP) at $t =$ 0 to 1 by taking interval 0.2, using

(i) Euler's method, and

(6 marks)

(ii) Modified Euler's method.

(6 marks)

(iii) Hence, compare the effectiveness of both methods if the exact solution is $v(t) = (t^3 + 3t^2 + 3t + 1)e^{-t}$.

(3 marks)

(c) The following methods can be used to solve an ODE $x' = f(t, x)$.

(i) $x_{n+1} = x_n + \frac{1}{2}[f(t_n, x_n) + f(t_{n+1}, x_n + f(t_n, x_n)h)]h$.

(ii) $x_{n+1} = x_n + \frac{1}{2}[3f(t_n, x_n) - f(t_{n-1}, x_{n-1})]h$.

(iii) $x_{n+1} = x_n + \frac{1}{2}[f(t_n, x_n) + f(t_{n+1}, x_{n+1})]h$.

For each method, determine if it has the following properties: single-step method, implicit method and Runge-Kutta type method.

(6 marks)

- END OF QUESTIONS -

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FORMULA

Nonlinear equations

Newton-Raphson method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \ldots$

System of linear equations

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, \quad \forall i = 1, 2, 3, \ldots, n$

Interpolation

Lagrange interpolation : $P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$ for $k = 0, 1, 2, 3, \ldots, n$ with $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \dfrac{(x - x_j)}{(x_i - x_j)}$

Eigen value

Gerschgorin's theorem:

$$r_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|, \qquad D_i = \{z \in \mathbf{C} : |z - a_{ii}| \leq r_i\}, \qquad \lambda_k \in \bigcup_{i=1}^{n} D_i \quad \text{for } k = 1, 2, \ldots, n$$

Power Method: $\mathbf{v}^{(k+1)} = \dfrac{1}{m_{k+1}} A \mathbf{v}^{(k)}, \qquad k = 0, 1, 2, \ldots$

Shifted Power Method: $\mathbf{A}_{\text{shifted}} = \mathbf{A} - l_{\text{Largest}} \mathbf{I}, \ \lambda_{\text{smallest}} = \lambda_{\text{Shifted}} + l_{\text{Largest}}$

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Numerical differentiation and integration

Differentiation:

First derivatives:

5-point difference: $f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Second derivatives:

5-point difference: $f''(x) \approx \dfrac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

Integration:

Gauss Quadrature:

for $\int_a^b f(x)dx,\quad x=\dfrac{(b-a)t+(b+a)}{2}$

2 points $\int_{-1}^{1} f(x)dx \approx g(-\frac{1}{\sqrt{3}})+g(\frac{1}{\sqrt{3}})$

3 points $\int_{-1}^{1} f(x)\,dx \approx \frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right)+\frac{8}{9}g(0)+\frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)$

Ordinary differential equations

Initial value problems:

Euler's Method: $y_{i+1}=y_i+h\,f(x_i,y_i)$

Modified Euler's method : $y_{i+1}=y_i+\frac{1}{2}k_1+\frac{1}{2}k_2$

where $k_1=hf(x_i,\, y_i)$ $\qquad k_2=hf(x_i+h,\, y_i+k_1)$