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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	NUMERICAL METHODS II
COURSE CODE	:	BWA 32403
PROGRAMME CODE	:	BWA / BWQ
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

TERBUKA

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 Boundary value problem

$$y'' = p(x)y' + q(x)y + r(x) \quad , \quad a \le x \le b, \quad y(a) = \alpha \ , \quad y(b) = \beta \qquad (1)$$

can be solved by considering a set of two initial value problems

$$y'' = p(x)y' + q(x)y + r(x) \quad , \quad a \le x \le b, \ y(a) = \alpha \ , \ y'(a) = 0 \qquad (2)$$

and

$$y'' = p(x)y' + q(x)y \quad , \quad a \le x \le b, \ y(a) = 0 \ , \ y'(a) = 1 \qquad (3)$$

Let $y_1(x)$ and $y_2(x)$ as solutions of (2) and (3) respectively, show that

$$y(x) = y_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y_2(x), \qquad y_2(b) \ne 0$$

is the solution of (1).

(9 marks)

Q2 Derive the first derivative of two point forward difference and two point central difference, and second derivative three point central difference formulas for function $f(x, y)$.

(12 marks)

Q3 (a) Solve the following set of ordinary differential equations using 4th order Runge-Kutta method

$$\frac{dT}{dx} = z, \qquad (1)$$

$$\frac{dz}{dx} = T - 1, \qquad (2)$$

subject to initial conditions $T(0) = 1$ and $z(0) = e$. Solve the system from $x = 0$ to $x = 1.0$ with $\Delta x = 0.5$. (Use up to 3 decimal places)

(15 marks)

(b) Repeat **Q3**(a) by assuming the initial condition of z is $3e$.

(10 marks)

Q4 The conservation of steady heat for long, thin, non-insulated uniform rod positioned between two bodies of constant but different temperature T_1 and T_2 where $T_1 < T_2$ and $T_2 > T_a$ is given by the following equation:

$$\frac{d^2T}{dx^2} + k(T_a - T) = 0,$$

where k is a heat transfer coefficient that parameterizes the rate of heat dissipation to the surrounding air and T_a is the temperature of the surrounding air for 1m rod with $k = T_a = T_1 = 1$ and $T_2 = e^2$.

(a) Plot the figure to visualize the problem.

(2 marks)

(b) Given that the analytical solution of the problem is $T(x) = e^{1+x} - e^{1-x} + 1$, solve the system using shooting method from $x = 0$ to $x = 1.0$ with $\Delta x = 0.5$. (Hint: Use **Q1**, **Q3** and linear interpolation)

(13 marks)

Q5 Consider the boundary value problem (BVP)

$$a\frac{d^2T}{dx^2} + b\frac{dT}{dx} + cT + d = 0$$

where a, b, c and d are any nonzero constants and $a = 1, b = 0,\ c = -1,\ d = 1$. The analytical solution of BVP subject to boundary conditions $T(0) = 1$ and $T(1) = e^2$ is given by $T(x) = ee^x - ee^{-x} + 1$.

a) Solve BVP using finite difference method from $x = 0$ to $x = 1.0$ with subinterval

(i) $M = 2$

(5 marks)

(ii) $M = 3$

(7 marks)

b) Compute the percent error in your numerical calculation.

(2 marks)

Q6 Evaluate the following double integral using the trapezoid rule with subinterval $n = 4$.

$$\int_0^4 \int_{-2}^2 (x^2 - 3y^2 + xy^3)\,dy\,dx$$

(15 marks)

Q7 Based on Fourier's law of heat conduction, the temperature in a rod of unit length is given by the simplest parabolic equation in one-dimensional unsteady partial differential equation

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2},$$

where k is the coefficient of thermal diffusivity, $T(x,t)$ is the temperature, t is the time $t > 0$, and x is the length of the rod. Here, $0 < x < 1$. The initial temperature of the rod is $T(x,0) = x^2$ for $0 < x < 1$, and the boundary conditions are $T(0,t) = 0$ and $T(1,t) = 1$ for $t > 0$. Taking $\Delta x = 0.2$, $t = 0.1$ and $\Delta t = 0.02$ find the difference equation of the problem using implicit method and write the tridigonal matrix that represent the set of simultaneous equations. (without solution)

(10 marks)

– **END OF QUESTIONS** –

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Fourth-order Runge-Kutta Method for $\frac{dy}{dx} = f(x, y)$ is given by

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4),$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$