UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATION
COURSE CODE	:	BWC 10603
PROGRAMME CODE	:	BWC
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A** AND CHOOSE **ONE (1)** IN **SECTION B**

TERBUKA

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

SECTION A

Q1 (a) (i) Show that $y(x) = x^{-3/2}$ is a solution to $4x^2 y'' + 12xy' + 3y = 0$.

(5 marks)

(ii) Show that the solution in **Q1(a)(i)** also satisfies the initial condition, which is given by $y(4) = \dfrac{1}{8}$ and $y'(4) = -\dfrac{3}{64}$.

(6 marks)

(b) (i) The modeling of first order linear differential equation that represents physical situation of falling object caused by gravitational force and air resistance can be described by

$$\frac{dv}{dt} + 0.2v - 9.8 = 0$$

Find out the implicit solution.

(7 marks)

(ii) If the initial value problem is given by $v(0) = 48$, write the explicit solution for **Q1(b)(i)**.

(3 marks)

(iii) Solve the following differential equation using separable equation.

$$\frac{dy}{dx} = 6xy^2$$

(4 marks)

Q2 (a) Solve $(1 + x^2) dy + 2xy dx = 0$ using exact equation.

(12 marks)

(b) Outline the following initial value problem using Laplace transformation.

$$2y'' + 3y' - 2y = te^{-2t}, \qquad y(0) = 0 \text{ and } y'(0) = -2$$

(13 marks)

Q3 (a) Solve the following second order differential equation

(i) $y'' - 6y' - 2y = 0$.

(5 marks)

(ii) $4y'' + 24y' + 37y = 0$ with IVP $y(\pi) = 1$ and $y'(\pi) = 0$.

(10 marks)

(b) Outline the general and particular solution of nonhomogeneous differential equation $y'' - 4y' - 12y = te^{4t}$.

(10 marks)

SECTION B

Q4 (a) A 4.905 N object stretches a spring to 0.273 m by itself. There is no damping and no external forces acting on the system. The spring is initially displaced 0.137 m upwards from its equilibrium position and with an initial velocity of 1 m/s downward. Analyze the displacement at any time, $u(t)$.

(15 marks)

(b) Take the spring and mass system from **Q4(a)** and a damper is attached to it that will exert a force of 12 N when the velocity is 2 m/s. Analyze the displacement at any time, $u(t)$.

(10 marks)

Q5 (a) A 50 kg object is shot from a cannon straight up with an initial velocity of 10 m/s off a bridge that is 100 m above the ground as shown in **Figure Q5(a)**. If the term of air resistance is given by $5v$, analyze the velocity of the mass when it hits the ground.

(12 marks)

(b) A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of 0.2 (1 + cos (t)) lbs/gal. If a well-mixed solution leaves the tank at a rate of 6 gal/hr, analyze the amount of salt in the tank when it overflows?

(13 marks)

– **END OF QUESTIONS** –

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FINAL EXAMINATION

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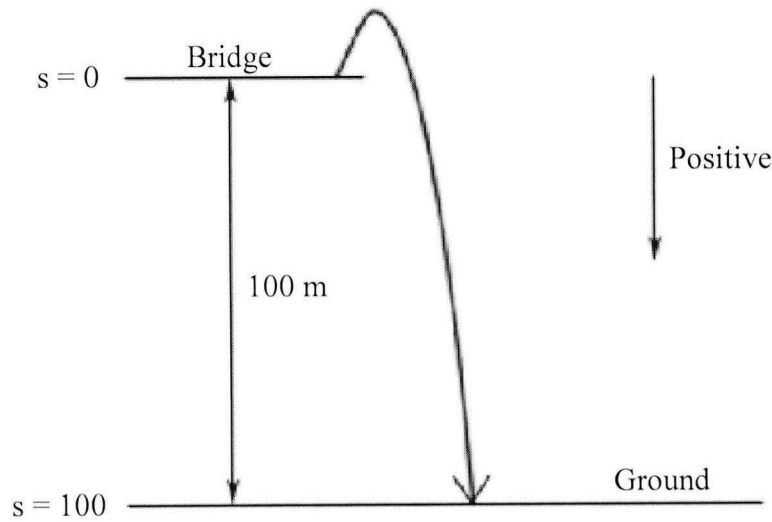COURSE NAME   : ORDINARY DIFFERENTIAL EQUATION   COURSE CODE   : BWC 10603

Figure Q5(a)