UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWA 20303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019/JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) (i) State the difference between the ordinary differential equations and partial differential equations in terms of the independent variable.

(2 marks)

(ii) Determine the order, the degree and the independent variable of the following differential equation,

$$\left(\frac{d^4 r}{d\theta^4}\right)^3 + \left(\frac{dr}{d\theta}\right)^5 + r^3 = x .$$

(3 marks)

(b) Solve the first order differential equation given as

$$(e^{-y} + \cos(x-y) + 2x)\,dx + (-xe^{-y} - \cos(x-y) - 1)\,dy = 0 .$$

(7 marks)

(c) Water is heated to a boiling point temperature 120^{0}C. It is then removed from the burner and kept in a room of 30^{0}C temperature. Assume that there is no change in the temperature of the room and the temperature of the hot water is 110^{0}C after 3 minutes.

(i) Find the temperature of the water after 6 minutes.

(4 marks)

(ii) Predict the approximate duration in which the water will cool down to the room temperature. Calculation has to be shown.

(4 marks)

Q2 (a) A spring is stretched 0.49 m ($\Delta\ell$) when a 6 kg mass (m) is attached. The weight is then pulled down for an additional 0.8 m and released with an upward velocity of 10 ms^{-1}. Neglect the damping constant, c. If the general equation describing the spring-mass system is

$$m\frac{d^2 y}{dt^2} + c\frac{dy}{dt} + ky = 0,$$

find an equation for the position of the spring at any time t.

(Hints : Weight, $W = mg$, $k = \dfrac{W}{\Delta\ell}$, $g \approx 9.8$ ms^{-2})

(10 marks)

(b) Solve the differential equation,

$$y'' - 2y' + y = \frac{e^x}{x}$$

by using the variation of parameters method.

(10 marks)

Q3 (a) By using substitution $u = 3t$, show that

$$\int_0^{\infty} (3t^2 + t + 2)\,\delta(3t-1)\,dt = \frac{1}{9}\int_0^{\infty} (u^2 + u + 6)\,\delta(u-1)\,du.$$

Hence, evaluate the integrals.

(5 marks)

(b) Prove that $L\left\{\frac{1}{2}\sin 2t - \cos 2t\right\} = \frac{8}{(s^2+4)^2}$.

(6 marks)

(c) A damped force oscillation is given by

$$y'' + 2y' + 5y = e^{-t}\sin 2t, \qquad y(0) = 0, \quad y'(0) = 1.$$

By Laplace Transform, find $y(t)$.

(9 marks)

Q4 (a) Produce the solution of $y'' + (\cos x)y = 0$ by assuming that the solution has the form of $y = \sum_{m=0}^{\infty} c_m x^m$.

(10 marks)

(b) By using an appropriate power series method, determine the solution to the given equation up to x^3 only.

$$y' + e^{-x}y = x^3, \quad y(0) = 3.$$

(10 marks)

Q5 (a) Solve the given the system of first order differential equations

$$y'_1 = 4y_1 + 2y_2,$$
$$y'_2 = 3y_1 + 3y_2.$$

(8 marks)

(b) By using the Laplace transform, solve the following system of linear differential equations

$$x' + x - y = 0$$
$$y' - x + y = 2$$

subject to conditions $x(0) = 1, \quad y(0) = 2$.

(12 marks)

– **END OF QUESTIONS** –

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$ or $a\ddot{y}+b\dot{y}+cy=0$ or $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay''+by'+cy=f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x)=A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c=Ay_1+By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y=y_c+y_p, \quad \text{and} \quad y_p=uy_1+vy_2,$$

$$\text{where } u=-\int\frac{y_2f(x)}{aW}dx, \qquad v=\int\frac{y_1f(x)}{aW}dx \quad \text{and} \quad W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}=y_1y_2'-y_2y_1'.$$

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Shift property of Impulse Function : $\int_0^\infty f(t)\delta(t-a)dt = f(a)$

Representation of Functions in Power Series :

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots, \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots, \quad e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\ldots$$

Maclaurin series :

$$y(x) = \sum_{m=0}^{\infty} y^m(0)\frac{x^m}{m!} = y(0) + y'(0)x + \frac{1}{2!}y''(0)x^2 + \frac{1}{3!}y'''(0)x^3 + \ldots\ldots\ldots\ldots$$

Taylor series, at $x = a$:

$$y(x) = \sum_{m=0}^{\infty} y^m(a)\frac{(x-a)^m}{m!} = y(a) + y'(a)(x-a) + \frac{1}{2!}y''(a)(x-a)^2 + \frac{1}{3!}y'''(a)(x-a)^3 + \ldots\ldots\ldots\ldots$$

Trigonometric identities : $\cos 2x = 1 - 2\sin^2 x$