UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	PROBABILITY AND STATISTICS II
COURSE CODE	:	BWB 10503
PROGRAMME CODE	:	BWA / BWQ
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Suppose the probabilities are 0.89, 0.09 and 0.02 that the finish on a new car will be rated acceptable, easily repairable or unacceptable. Compute the probability that among 20 cars painted one morning, 17 have acceptable finishes, 2 have repairable finishes and 1 finish is unacceptable.

(4 marks)

(b) A shipment of 120 burglar alarms contains 5 that are defective. If 3 of these alarms are randomly selected and shipped to a customer,

(i) identify an appropriate distribution form for this shipment.

(1 mark)

(ii) calculate the probability that the customer will get at least two bad units.

(5 marks)

(c) The amount of time that a surveillance camera will run without having to be reset is a random variable with mean 50 days. Obtain the probability that such a camera will have to be reset in less than 20 days.

(6 marks)

(d) Suppose that the proportion of defectives shipped by the vendor, which varies somewhat from shipment to shipment, may be looked upon as a random variable having the beta distribution with $\alpha = 1$ and $\beta = 4$.

(i) Calculate the average proportion of defectives in a shipment from this vendor.

(2 marks)

(ii) Find the probability that a shipment from this vendor will contain 25% or more defectives.

(7 marks)

Q2 (a) Hard disks for computers must spin evenly and one departure from level is called the roll. The roll for any disk can be modelled as a random variable having mean 0.225 mm and standard deviation 0.06 mm. The sample mean roll $\overline{X}$ will be obtained from a random sample of 36 disks.

(i) Find the mean and standard deviation of the sampling distribution of $\overline{X}$.

(3 marks)

(ii) Calculate the probability that $\overline{X}$ will lie between 0.21 and 0.245 mm.

(7 marks)

(b) Consider two samples

$$W_1 = \frac{X_1 + X_2 + \cdots + X_6}{6} \text{ and } W_2 = \frac{5X_1 - X_6 + 2X_4}{6}.$$

(i) Determine whether they are both an unbiased estimator for μ or not.

(7 marks)

(ii) If yes, recommend the most efficient estimator.

(8 marks)

Q3 A copy shop records that in 61 cases, the cartridge for the copy machine lasted has an average of 18300 copies with a standard deviation of 2800 copies.

(a) Obtain a 95% confidence interval for the population mean number of copies before a new cartridge is needed for the copy machine.

(8 marks)

(b) Find the margin of error with a 98% probability if $\overline{x} = 18300$ is used as a point estimate of the true population mean number of copies.

(7 marks)

(c) Compute a 90% confidence interval for the population standard deviation of copies before a new cartridge is needed for the copy machine.

(10 marks)

Q4 (a) A producer of extruded plastic products finds that his mean daily inventory is 1250 pieces. A new marketing policy has been put into effect and it is desired to test the null hypothesis that the mean daily inventory is still the same. Specify the null hypothesis and the alternative hypothesis in each of the following cases

(i) If it is desired to know whether or not the new policy changes the mean daily inventory.

(2 marks)

(ii) If it is desired to demonstrate that the new policy actually reduces the mean daily inventory.

(2 marks)

(iii) If the new policy will be retained so long as it cannot be shown that it actually increases the mean daily inventory.

(2 marks)

(b) The manufacturing of large liquid crystal displays (LCD's) is difficult. Some defects are minor and can be removed; others are un-removable. The number of un-removable defects for two samples are measured yielding the following results.

Sample 1 :	$n_1 = 16$	$\bar{x}_1 = 2.98$	$s_1 = 0.71,$
Sample 2 :	$n_2 = 13$	$\bar{x}_2 = 2.35$	$s_2 = 0.55.$

(i) Assume the populations to be approximately normal with equal variances, conduct a hypothesis test with the intent of showing that the difference means of these two samples is significant at 0.02 level of significance.

(15 marks)

(ii) Based on your conclusion in **Q4 (b) (i)**, what error could you have made? Explain in the context of the problem.

(4 marks)

- END OF QUESTIONS -

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FORMULA

$$f(x)=\begin{cases}\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

$$f(x)=\begin{cases}\dfrac{1}{\Gamma(\alpha)}k^{\alpha}x^{\alpha-1}e^{-kx} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

$$f(x)=\begin{cases}\dfrac{1}{b-a} & a\le x\le b\\ 0 & \text{elsewhere}\end{cases}$$

$$f(x)=\begin{cases}\dfrac{1}{\beta}e^{-x/\beta} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

$$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$$

$$f(x)=p(1-p)^{x-1}$$

$$\Pr(X=x)=\frac{{}^{r}C_{x}\,{}^{N-r}C_{n-x}}{{}^{N}C_{n}}$$

$$E(X)=\alpha\beta \qquad Var(X)=\alpha\beta^2$$

$$E(X)=\frac{a+b}{2} \qquad Var(X)=\frac{1}{12}(b-a)^2$$

$$E(X)=\frac{1}{p} \qquad Var(X)=\frac{1-p}{p^2}$$

$$E(X)=\beta \qquad Var(X)=\beta^2$$

$$E(X)=\frac{nr}{N} \qquad E(X)=\frac{\alpha}{\alpha+\beta}$$

$$Var(X)=\frac{r(N-r)n(N-n)}{N^2(N-1)}$$

$$Var(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$Z=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}}\sim Z_\alpha$$

$$Z=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}}\sim Z_\alpha$$

$$T=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{1}{n}(s_1^2+s_2^2)}}\sim T_\alpha(v)$$

$$T=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{S_p\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}\sim T_\alpha(v)$$

$$T=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}}\sim T_\alpha(v)$$

$$\frac{s_1^2}{s_2^2}\frac{1}{f_{\alpha/2}(v_1,v_2)}<\frac{\sigma_1^2}{\sigma_2^2}<\frac{s_1^2}{s_2^2}f_{\alpha/2}(v_2,v_1)$$

$$Z=\frac{x-\mu}{\sigma}\sim Z_\alpha$$

$$Z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\sim Z_\alpha$$

$$T=\frac{\bar{x}-\mu}{s/\sqrt{n}}\sim T_\alpha(v=n-1)$$

$$\chi^2=\frac{(n-1)s^2}{\sigma^2}\sim\chi^2_\alpha(v=n-1)$$

$$F=\frac{s_1^2}{s_2^2}\sim F_\alpha(v_1=n_1-1,v_2=n_2-1)$$

$$S_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

$$v=\frac{(s_1^2/n_1+s_2^2/n_2)^2}{\dfrac{(s_1^2/n_1)^2}{n_1-1}+\dfrac{(s_2^2/n_2)^2}{n_2-1}}$$

$$v=n_1+n_2-2$$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,v}}<\sigma^2<\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,v}}$$