UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	QUEUING
COURSE CODE	:	BWA 40503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 A doctor in a clinic can handle patients according to exponential distribution with 10 minutes per patient. The arrival of the patients to the clinic is according to Poisson distribution with the arrival rate of 10 patients per hour. The clinic only provide 3 seats for waiting customers. Determine

a) the maximum number of the patients in the clinic.

(1 mark)

b) the probability of n customers in the system for all possible n.

(12 marks)

c) the average time (in minute) a patient spend in the clinic.

(7 marks)

Q2 Iman Supermarket operates with four check-out counters. The manager uses the schedule in **Table Q2** to determine the number of counters in operation, depending on the number of customers in store.

Table Q2: Number of customers versus the Number of counters

No. of customers in store	No. of counters in operation
1 to 3	1
4 to 6	2
7 to 8	3
More than 8	4

Customers arrive in the counters area according to a Poisson distribution with a mean rate of 10 customers per hour. The average check-out time per customer is exponential with mean 12 minutes.

(a) Derive the steady-state probabilities, P_n in terms of P_0 for all possible n.

(13 marks)

(b) Find the probability of number of counters open, $P(X)$ for $X = 0, 1, 2, 3$ and 4.

(18 marks)

(c) Find the expected number of idle counters.

(4 marks)

Q3 Customers arrive at a post office according to a Poisson distribution at the rate of 17.5 customers per hour. Each clerk can handle an average of 10 customers per hour. The cost of hiring a new clerk in the facility is RM12 an hour. The cost of lost service per waiting customers per hour is approximately Rm50. The post office can hire not more than 5 clerks.

(a) Decide whether one clerk can handle the arriving customers. Give the reason.

(2 marks)

(b) Find the probability of no customer for case of $2, 3, 4$ and 5 number of servers respectively.

(19 marks)

(c) Determine the que length for the number of servers, $c = 2, 3, 4$ and 5.

(13 marks)

(d) By using the cost model, determine the optimal number of clerks and hence, the minimum cost for the post office.

(11 marks)

- END OF QUESTIONS -

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FORMULAS

Geometric Series	$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}\ ;\ \lvert x\rvert \leq 1$	$\sum_{i=0}^{s} x^i = \frac{1-x^{s+1}}{1-x}$
General Queuing	$L_s = \sum_{n=1}^{\infty} np_n = \lambda_{\text{eff}} W_s = L_q + \frac{\lambda_{\text{eff}}}{\mu}$ $L_q = \sum_{n=c+1}^{\infty} (n-c)p_n = \lambda_{\text{eff}} W_q$ $W_s = W_q + \frac{1}{\mu}$	
Single Server	$P_0 = 1-\rho$	$L_s = \frac{\rho}{1-\rho}$
Single Server with N Customers	$P_0 = \begin{cases} \frac{(1-\rho)}{1-\rho^{N+1}}, & \rho \neq 1 \\ \frac{1}{N+1}, & \rho = 1 \end{cases}$	$P_n = \begin{Bmatrix} \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \frac{1}{N+1}, & \rho = 1 \end{Bmatrix} n = 0,1,\ldots,N$
Multiple-Server	$P_0 = \left\{\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!}\left(\frac{1}{1-\frac{\rho}{c}}\right)\right\}^{-1},\quad \frac{\rho}{c} < 1$	$P_n = \begin{Bmatrix} \frac{\rho^n}{n!}P_0, & n < c \\ \frac{\rho^n}{c!\,c^{n-c}}P_0, & n \geq c \end{Bmatrix}$
	$L_q = \frac{\rho^{c+1}}{(c-1)!\,(c-\rho)^2} P_0$	
Multiple-Server with N Customers	$P_0 = \begin{cases} \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c\left(1-\left(\frac{\rho}{c}\right)^{N-c+1}\right)}{c!\left(1-\frac{\rho}{c}\right)}\right]^{-1}, & \frac{\rho}{c} \neq 1 \\ \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!}(N-c+1)\right]^{-1}, & \frac{\rho}{c} = 1 \end{cases}$	
	$P_n = \begin{Bmatrix} \frac{\rho^n}{n!}P_0, & 0 \leq n < c \\ \frac{\rho^n}{c!\,c^{n-c}}P_0, & c \leq n \leq N \end{Bmatrix}$	
Self-Service Model	$P_0 = e^{-\rho}$ $P_n = \frac{e^{-\rho}\rho^n}{n!}$	$L_s = \rho$

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Machine Servicing Model	$$P_0 = \left(\sum_{n=0}^{R} C_n^K \rho^n P_0 + \sum_{n=R+1}^{K} \frac{C_n^K \rho^n n!}{R!\, R^{n-R}} \right)^{-1}$$
	$$P_n = \begin{cases} C_n^K \rho^n P_0, & 0 \le n \le R \\ \dfrac{C_n^K \rho^n n!}{R!\, R^{n-R}} P_0, & R < n \le K \end{cases}$$
	$$L_s = \sum_{n=0}^{K} n p_n$$

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