UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103
PROGRAMME	:	2 BFF
EXAMINTION DATE	:	JUNE 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Given $f(x,y)=\dfrac{3xy^4}{2x^2+5y^8}$.

(i) Determine whether or not $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exist, by letting $(x,y)\to(0,0)$ along any straight line $y=mx$ and the curve $x=y^4$.

(ii) Is the function $f(x,y)$ continuous at (0, 0)?

(13 marks)

(b) The length l, width w, and height h of a box change with time. At a certain instant, the dimensions are $l=1$ m and $w=h=2$ m, and l and w are increasing at a rate of 2 m/s while h is decreasing at a rate of 3 m/s. At that instant, find the rates at which the following quantities are changing.

(i) The volume.

(ii) The surface area.

(12 marks)

Q2 (a) By using polar coordinate, evaluate $\iint_R (x+y)\,dA$ where R is the region in the first quadrant lying inside the disc $x^2+y^2\le 9$ and under the line $y=x$.

(5 marks)

(b) Evaluate the following integral by changing to spherical coordinates.

$$\int_0^2\int_0^{\sqrt{4-y^2}}\int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\;dz\,dx\,dy$$

(7 marks)

(c) A lamina has a shape of triangle with vertices $(0,0)$, $(0,2)$ and $(2,0)$. If the density is $\delta(x,y)=xy$, find the centre of mass.

(13 marks)

Q3 (a) The position vector of a particle is

$$\mathbf{r}(t)=\sqrt{t}\,\mathbf{i}+(2t+4)\,\mathbf{j}.$$

(i) Sketch the graph of $\mathbf{r}(t)$ by indicating the direction of the vector.

(ii) Find the velocity, speed and acceleration of the particle at $t=2$.

(13 marks)

(b) Given the vector-valued function

$$\mathbf{r}(t)=\cos 5t\,\mathbf{i}+\sin 5t\,\mathbf{j}+2t\,\mathbf{k}.$$

Find its unit tangent vector, $\mathbf{T}(t)$, principal unit normal vector, $\mathbf{N}(t)$ and curvature, κ at $t=\dfrac{\pi}{2}$.

(12 marks)

Q4 (a) Use Green's theorem to rewrite and evaluate $\oint_C (x^2 + y^3)dx + 3xy^2 dy$, where C consists of the portion of $y = x^2$ from $(2,4)$ to $(0,0)$, followed by the line segments from $(0,0)$ to $(2,0)$ and from $(2,0)$ to $(2,4)$.

(4 marks)

(b) Find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)\mathbf{i} + (\sin y + x^3)\mathbf{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in counterclockwise direction.

(10 marks)

(c) Given that $\mathbf{F}(x, y) = 2xy^3\mathbf{i} + (2 + 3x^2y^2)\mathbf{j}$.

(i) Show that $\mathbf{F}$ is a conservative vector field on the entire plane xy - plane.

(ii) Find the potential function.

(11 marks)

Q5 (a) Use the Divergence Theorem to find the outward flux of the vector field $\mathbf{F}(x, y) = x^3\mathbf{i} + y^5\mathbf{j} + z^3\mathbf{k}$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.

(13 marks)

(b) By means of Stoke Theorem, evaluate the line integral $\int_C \mathbf{F}.dr$ for $\mathbf{F}(x, y, z) = -y\mathbf{i} + x^2\mathbf{j} + z^3\mathbf{k}$ where C is the intersection of a circular cylinder $x^2 + y^2 = 4$ and the plane $x + z = 3$, oriented so that it is traversed counterclockwise when viewed from the positive z- axis.

(12 marks)

- END OF QUESTION -

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Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$,

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$, and

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

Directional derivative: $D_{\mathbf{u}} f(x,y) = \left(f_x\mathbf{i} + f_y\mathbf{j}\right)\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

the **curl** of

$$\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) dA$

Gauss Theorem: $\iint_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$

Arc length

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If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length**

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt$$

Tangent Plane

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Extreme of two variable functions

$$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\, dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\, dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\, dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m} \right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x,y)\, dA$, (ii) $I_x = \iint_R y^2 \delta(x,y)\, dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\, dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\, dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\, dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\, dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\, dV$

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Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$

Moment inertia

(i) about x-axis: $I_x = \iiint\limits_G (y^2 + z^2)\delta(x, y, z)\,dV$

(ii) about y-axis: $I_y = \iiint\limits_G (x^2 + z^2)\delta(x, y, z)\,dV$

(iii) about z-axis: $I_z = \iiint\limits_G (x^2 + y^2)\delta(x, y, z)\,dV$