CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003 / BWM 10203
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) By using homogenous equations method, solve the differential equation

$$\frac{dy}{dx} = -\frac{x^2 + y^2}{xy}$$

given that $y = 2$ when $x = 2$

(9 marks)

(b) The remaining soup need to be put into the fridge. The soup has just boiled at 100°C. The fridge could not accommodate a pot of soup if it is any warmer than 20°C. The pot need to cooling down below 20°C by cooling the pot in a sink full of a cold water. The cold water was kept running, thus the temperature of water in the sink was roughly constant at 5°C. In ten minutes, the temperature drop to 60°C. Determine the duration for a pot of soup can be put into the fridge.

(11 marks)

Q2 (a) Given a non-homogenous second order differential equation

$$y'' + 6y' + 10y = 2\,sin2x$$

Fine the general solution for the equation by using undetermined coefficient.

(10 marks)

(b) A spring with a 3 kg mass is held stretched 0.6 m beyond its natural length by a force of 20 N. If the spring begins at its equilibrium position but a push gives it an initial velocity of 1.2 m/s, find the position of the mass after 5 seconds.

(10 marks)

Q3 (a) Find the Laplace transform for each of given function

$$f(t) = 2t^4 - e^{-4t}$$

(4 marks)

(b) Find the inverse Laplace transforms of the following expressions.

(i) $\frac{3}{9s^2+4}$

(ii) $\frac{2s}{16s^2+9}$

(6 marks)

(c) By using Laplace transform method, solve the initial value problem

$$y'' + 4y = e^{-t}, y(0) = 2, y'(0) = 1$$

(10 marks)

Q4 (a) For the following power series determine the interval and radius of convergence.

$$\sum_{n=0}^{\infty} \frac{4^{1+2n}}{5^{n+1}} (x+3)^n$$

(10 marks)

(b) Use one of the Maclaurin Series derived to determine the Maclaurin Series for $f(x) = \sin(2x)$.

(10 marks)

Q5 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} -2, & -\pi \le x < 0, \\ 2, & 0 \le x < \pi. \end{cases}$$

$$f(x) = f(x + 2\pi).$$

which it can be written as a Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

(a) Sketch the graph of $f(x)$ over $-2\pi \le x \le 2\pi$.

(2 marks)

(b) Determine whether the function is even, odd or neither.

(2 marks)

(c) For $n = 1, 2, 3, \ldots$, show that

(i) $a_0 = 0$.

(ii) $a_n = 0$.

(7 marks)

(d) Calculate the Fourier coefficient, b_n.

(5 marks)

(e) Find the Fourier series.

(4 marks)

-END OF QUESTION-

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$ $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Particular Integral of $ay'' + by' + cy = f(x)$ **: Method of Undetermined Coefficients**

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$ **: Method of Variation of Parameters**

Wronskian	Parameter	Solution
$y_h = Ay_1 + By_2$ $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ $= y_1 y_2' - y_2 y_1'$	$u_1 = -\int \dfrac{y_2 f(x)}{W}dx, \quad u_2 = \int \dfrac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt,\quad s>0.$

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Maclaurin Series

$$f(x) = \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

$$= \sum_{p=0}^{n} \frac{f^{(p)}(0)}{p!}x^p$$

Fourier Series

Fourier series expansion of periodic function with period $2L$	**Fourier half-range series expansion**
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$
where	where
$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$	$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$
$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$