UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103 / BWM 20403
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ONE (1)** QUESTION IN **SECTION A** AND **ALL** QUESTIONS IN **SECTION B**.

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

SECTION A: ANSWER ONE (1) QUESTION ONLY

Q1 (a) Evaluate the divergence and curl of **F** for the following vector fields.

(i) $\mathbf{F}(x, y, z) = xyz\,\mathbf{i} + y\,\mathbf{j} + x\,\mathbf{k},$ (1, 2, 3)

(ii) $\mathbf{F}(x, y, z) = e^{-x}\sin y\,\mathbf{i} + e^{-x}\cos y\,\mathbf{j} + \mathbf{k},$ (-2, 2, -2)

(12 marks)

(b) Given $f(x, y, z) = x^2y - y^2z + xz^3$. Calculate the gradient at (1, 2, 1) and the directional derivative at (1, 2, 1) in the direction of (3, 1, 3).

(5 marks)

(c) Use Green Theorem to evaluate $\oint_c y^3\,dx + (x^3 + 3xy^2)\,dy$ where C is the boundary of circle $x^2 + y^2 = 9$ oriented counterclockwise.

(8 marks)

Q2 (a) Calculate the surface area of the portion of $x + 3y + z = 6$ in the first octant.

(5 marks)

(b) Use the divergence theorem to evaluate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, where S is the entire surface of the paraboloid $z = 3 + x^2 + y^2$ bounded by the planes $z = 3$ and $z = 7$, if $\mathbf{F} = 2x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and **n** is the outwards unit.

(10 marks)

(c) By using Stokes' theorem, evaluate $\oint_c \mathbf{F}\cdot d\mathbf{r}$ for the vector $\mathbf{F} = xz\,\mathbf{i} + xy^2\mathbf{j} + 3xz\,\mathbf{k}$ and the space curve C which is the intersection of the plane $x + z = 3$ and the cylinder $x^2 + y^2 = 4$, in the counterclockwise direction, when viewed from the positive z-axis.

(10 marks)

SECTION B: ANSWER ALL QUESTIONS

Q3 (a) Calculate the following limit:

$$\lim_{(x,y)\to(2,1)} \frac{x^2 - 2xy}{x^2 - 4y^2}$$

(5 marks)

(b) Formulate the equation of the tangent plane to $z = x^2 \cos(\pi y) - \frac{6}{xy}$ at (2, -1).

(10 marks)

(c) Given $z = x^2 + y^2$, where $x = \frac{1}{t^3}$ and $y = 2t^3$. Calculate $\frac{dz}{dt}$:

(i) By using $x = \frac{1}{t^3}$ and $y = 2t^3$ into $z = x^2 + y^2$.
(ii) By using chain rule.

(10 marks)

Q4 (a) Calculate the area of the regions enclosed by $y = x^2$ and $y = x + 2$ using double integral.

(5 marks)

(b) The region 'R' is a triangle and it is located on *xy*-plane. If the given planes is $4x + 2y + z = 4$, $x = 0$, $y = 0$ and $z = 0$, visualize and calculate the volume of the tetrahedron region bounded.

(12 marks)

(c) Calculate the volume of the sphere and below by cone if given data are $\rho = 1$ by the cone $\phi = \frac{\pi}{3}$

(8 marks)

Q5 (a) Determine the velocity, acceleration, and speed of a particle given by the position function $\mathbf{r}(t) = 2 \cos t\, \mathbf{i} + 3 \sin t\, \mathbf{j}$ at $t = 0$

(10 marks)

(b) Sketch the path of particle and draw the velocity and acceleration vectors for the specified value of *t*.

(15 marks)

- END OF QUESTIONS –

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Formulae

Implicit Partial Differentiation:

$$\frac{\partial z}{\partial x} = -\frac{f_x(x,y,z)}{f_z(x,y,z)} \quad or \quad \frac{\partial z}{\partial y} = -\frac{f_y(x,y,z)}{f_z(x,y,z)}$$

Small Increment, Estimating Value:

Total differential/approximate change, $\partial z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

Exact change, $dz = f(x_1, y_1) - f(x_0, y_0)$

Approximate value, $z = f(x_0, y_0) + dz$

Exact value, $z = f(x_1, y_1)$

Error: $|dz| = \left|\frac{\partial z}{\partial x}dx\right| + \left|\frac{\partial z}{\partial y}dy\right|$ and **Relative error:** $\left|\frac{dz}{z}\right| = \left|\frac{\partial z}{\partial x}\frac{dx}{z}\right| + \left|\frac{\partial z}{\partial y}\frac{dy}{z}\right|$

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\,dr\,d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\,dz\,dr\,d\theta$

Spherical coordinate: $x = \rho\cos\theta\sin\emptyset$, $y = \rho\sin\theta\sin\emptyset$, $z = \rho\cos\emptyset$, $x^2 + y^2 + z^2 = \rho^2$,
$0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$ and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

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Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$

Gauss Theorem: $\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iiint_G \nabla \bullet \mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \| \mathbf{r}'(t) \| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt$

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2 \delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

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Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m} \right)$

Moment inertia

(i) about x-axis: $I_x = \iiint\limits_G \left(y^2 + z^2\right)\delta(x, y, z)\,dV$

(ii) about y-axis: $I_y = \iiint\limits_G \left(x^2 + z^2\right)\delta(x, y, z)\,dV$

(iii) about z-axis: $I_z = \iiint\limits_G \left(x^2 + y^2\right)\delta(x, y, z)\,dV$