UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESI 2014/2015

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS IV
COURSE CODE	:	BFC 24203
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Approximate the value of $\sqrt[3]{7}$ using bisection method.

(9 marks)

(b) Use the Gauss-Seidel iteration method to approximate the solution of the following system of linear equations.

$$5x_1 - 2x_2 + 3x_3 = -1$$
$$2x_1 - x_2 - 7x_3 = 3$$
$$-3x_1 + 9x_2 + x_3 = 2$$

Continue the iterations until two successive approximations are identical when rounded to three decimal places.

(11 marks)

Q2 (a) For a function f, the divided-difference table from Newton's Divided-difference Method are given by

Table Q2(a)

$x_o = 2.1$	$f[x_0] = A$	$f[x_0, x_1] = -0.037$	$f[x_0, x_1, x_2] = -1.216$
$x_1 = 2.4$	$f[x_1] = B$	$f[x_1, x_2] = C$	
$x_2 = 2.6$	$f[x_2] = 0.381$		

(i) Determine the values of A, B and C in the above table.

(3 marks)

(ii) Interpolate the value of $f(2.5)$

(2 marks)

(b) Construct a natural cubic spline, if S(x) interpolates the data of (-2,2), (0,4) and (3,1). Then estimate $f(1.5)$.

(10 marks)

Q3 An object is dropped at various distances of x (m) is measured at time, t intervals of 0.2s. The data gathered is,

Table Q3

Time, t (sec)	0	0.2	0.4	0.6	0.8	1.0	1.2
Distance, x (m)	0.0	3.4	8.2	13.8	21.1	26.5	31.3

Estimate the object's velocity at time, t =1 using an appropriate difference formulas. Give the answers to 1 decimal place.

(10 marks)

Q4 (a) Given

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}.$$

By taking $v^{(0)} = (1 \quad 1 \quad 0)^T$, calculate the largest Eigen value and the corresponding Eigen vector by using a power method.

(5 marks)

(b) The initial value problem $y' = \frac{2y}{x} - xy^2$, $y(1) = 5$, has unique solution $y(x) = \frac{20x^2}{5x^4 - 1}$. Approximate the solution at $x = 2$ using the fourth order Runge-Kutta method with the same step size $h = 0.2$ and estimate the absolute error.

(5 marks)

Q5 (a) Given $f(x) = \sin x$. Approximate $\int_0^{\pi/4} \left(f'(x)\right)^2 \, dx$ by using

(i) 2-point Gauss-quadrature

(ii) 3-point Gauss-quadrature

(9 marks)

(b) Solve the boundary value problem, $y'' + xy = x^3 - \frac{4}{x}$, $1 \leq x \leq 2$, with boundary conditions, $4y(1) + y'(1) = 0$, and $3y(2) + 2y'(2) = 0$. Derive the system of linear equations in matrix-vector form by finite difference method (do not solve the system). Use $h = \Delta x = 0.2$.

(11 marks)

Q6 Given the heat equation $\pi \frac{\partial u}{\partial t} = \frac{4\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$ with boundary condition $u\,(0,t) = 0$ and $u\,(1,t) = t$ and the initial condition, $u\,(x,0) = x(1-x)$. By using explicit finite-difference method, solve the heat equation up to first level only $(t \leq 0.01)$ by taking $\Delta x = h = 0.2$ and $\Delta t = k = 0.01$

(15 marks)

Q7 A system of 10-cm thin rod subject to fixed boundary conditions with a continuous heat and finite-element representation consisting of four equal-length elements can be seen in **FIGURE Q7**. The boundary conditions of $T(0, t) = 40$ and $T(10, t) = 200$ and a uniform heat source of $f(x) = 10$.

The element equations are given as

$$\underbrace{\frac{1}{x_2 - x_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{\textit{Element stiffness matrix}}\{T\} = \underbrace{\begin{Bmatrix} -\dfrac{dT(x_1)}{dx} \\ \dfrac{dT(x_2)}{dx} \end{Bmatrix}}_{\substack{\textit{Boundary} \\ \textit{condition}}} + \underbrace{\begin{Bmatrix} \int_{x_1}^{x_2} f(x)N_1(x)dx \\ \int_{x_1}^{x_2} f(x)N_2(x)dx \end{Bmatrix}}_{\textit{External effects}}$$

with interpolation function, N

$$N_1 = \frac{x_2 - x}{x_2 - x_1} \text{ and } N_2 = \frac{x - x_1}{x_2 - x_1}$$

Develop the element equations for the rod by employing four equal-size elements of length = 2.5 cm. If the boundary conditions T follows $T = 2ax + b$, find the element equations in a and T.

(10 marks)

-END OF QUESTION-

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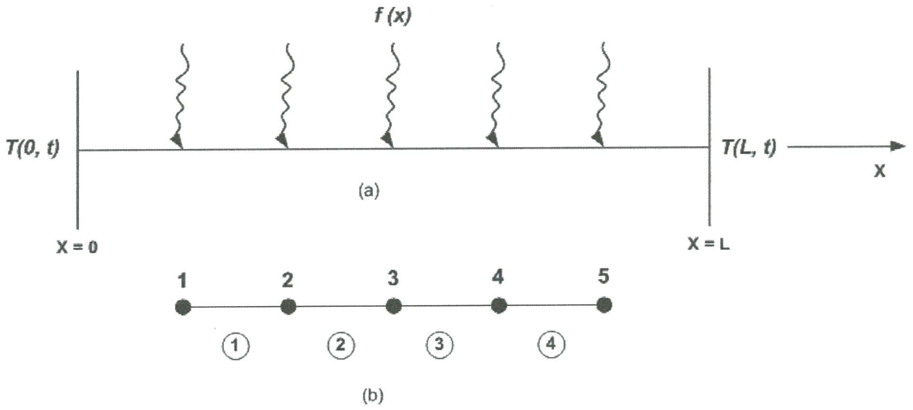

FIGURE Q7

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FORMULAE

Iteration formula for bisection method

$$c_i = \frac{a_i + b_i}{2}.$$

Gauss-Seidel iteration formula

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x^{(k)}}{a_{11}}$$

$$x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}$$

$$x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

Newton's Divided-difference method

$$P_n(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_0) + \ldots + f[x_0,x_1,\ldots,x_n](x-x_0)(x-x_1)\ldots(x-x_n)$$

Cubic spline interpolation

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1}-f_k}{h_k} \end{aligned}\right\} k = 0,1,2,3,\ldots, n-1,$$

$$b_k = 6(d_{k+1} - d_k),\ k = 0,1,2,3,\ldots, n-2,$$

$$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k,\ k = 0,1,2,3,\ldots, n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1}-x)^3 + \frac{m_{k+1}}{6h_k}(x-x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1}-x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x-x_k), \quad k = 0,1,2,3,\ldots, n-1$$

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FORMULAE

Method	Difference Formula
2 point forward	$\frac{f(x+h)-f(x)}{h}$
2 point backward	$\frac{f(x)-f(x-h)}{h}$
3 point central	$\frac{f(x+h)-f(x-h)}{2h}$
3 point forward	$\frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$
3 point backward	$\frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$
5 point	$\frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Fourth-order Runge-Kutta method

$$y_{i+1} = y_i + w_1 k_1 + w_2 k_2 + w_3 k_3 + w_4 k_4$$

where,

$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + c_2 h, y_i + a_{21} k_1)$$
$$k_3 = hf(x_i + c_3 h, y_i + a_{31} k_1 + a_{32} k_2)$$
$$k_4 = hf(x_i + c_4 h, y_i + a_{41} k_1 + a_{42} k_2 + a_{43} k_3)$$

2-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

3-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) + c_3 x_3 \approx \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right)$$