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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103 / BWM 20403
PROGRAMME	:	BACHELOR OF CIVIL ENGINEERING WITH HONOURS
DATE OF EXAMINATION	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FOUR (4)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Given that $f(x, y) = 2x^2 + xy - y^2$. By using the first principle, find the partial derivatives f_x and f_y.

(10 marks)

(b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if z is defined implicitly as a function of x and y in the equation;

$$2x(y + z) = x^2 + y^2 - z^2$$

(6 marks)

(c) The radius of a right circular cylinder is measured with an error of at most 4%, and the height is measured with and error of at most 8%. Approximate the maximum possible percentage error in the volume V calculated from these measurements.

(9 marks)

Q2 (a) By using cylindrical coordinates, find the volume of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = -5$.

(8 marks)

(b) By using spherical coordinates, find the volume of the solid located on top of a cone $z = \sqrt{x^2 + y^2}$ and inside a sphere $z^2 + y^2 + x^2 = 4z$.

(9 marks)

(c) By using double integrals, find the area of the regions enclosed by $y = \sin x, y = \cos x, x = 0, x = 45°$

(8 marks)

Q3 (a) Given that $\mathbf{F}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \cos t\,\mathbf{k}$ and $\mathbf{G}(t) = 3t\,\mathbf{i} - t\,\mathbf{j} + 2\mathbf{k}$. Calculate;

(i) $(\mathbf{F} + \mathbf{G})(t)$

(ii) $(\mathbf{F} \times \mathbf{G})(t)$

(4 marks)

(b) The position vector of a particle is

$$\mathbf{r}(t) = \cos 2t\,\mathbf{i} + 2\sin 2t\,\mathbf{j} + t^2\,\mathbf{k}$$

Find the velocity, speed, direction and acceleration of the particle at $t = \pi$

(6 marks)

(c) The equation of curve is given by $\mathbf{r}(t) = 5\sin t\,\mathbf{i} + 5\cos t\,\mathbf{j} + 12t\,\mathbf{k}$. Find the unit tangent vector **T**, principal unit vector **N**, curvature κ, radius of curvature ρ and binormal unit vector **B**.

(15 marks)

Q4 (a) Use Green's theorem to rewrite and evaluate $\oint(y - \cos x)\, dx + \sin x\, dy$, where C is the perimeter of the triangle formed by the lines $y = 0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$.

(6 marks)

(b) Given that $\mathbf{F}(x, y) = 2xy^3\mathbf{i} + (2 + 3x^2y^2)\mathbf{j}$.

(i) Show that **F** is a conservative vector field on the entire plane xy - plane.

(ii) Find its potential function.

(iii) Find the work done by the field on a particle that moves along the line segment from (1, 4) to (3, 1).

(19 marks)

Q5 (a) Use the Divergence Theorem and cylindrical coordinates to compute the outward flux of the vector field

$$\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j} + z^2\mathbf{k}$$

across the surface of the region that is enclosed by the circular paraboloid $z = 4 - x^2 - y^2$ and the planes $z = 0$.

(13 marks)

(b) Consider the vector field $\mathbf{F}(x, y) = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$, taking σ to be the portion of the paraboloid for which with upward orientation, and C to be positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane. Use Stokes' Theorem to evaluate the integral $\oint_C \mathbf{F}.\, d\mathbf{r}$

(12 marks)

- END OF QUESTION -

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Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\,dr\,d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\,dz\,dr\,d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$, and

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

the **curl** of

$$\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)dA$

Gauss Theorem: $\iint_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\in[a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\in[a,b]$, then the **arc length**

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$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt$$

Tangent Plane

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Extreme of two variable functions

$$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m} \right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2 \delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m} \right)$

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Moment inertia

(i) about x-axis: $I_x = \iiint_G \left(y^2 + z^2\right)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y = \iiint_G \left(x^2 + z^2\right)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z = \iiint_G \left(x^2 + y^2\right)\delta(x,y,z)\,dV$