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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS IV
COURSE CODE	:	BFC 24203/ BWM 30603
PROGRAMME	:	2 BFF/ 3 BFF/ 4 BFF 3 BDD/ 4 BDD
EXAMINATION DATE	:	DECEMBER 2014/ JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A** AND **TWO (2)** QUESTIONS IN **SECTION B.** B) ALL CALCULATIONS AND ANSWERS MUST BE IN **THREE (3) DECIMAL PLACES.**

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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SECTION A

Q1 (a) Given the heat equation

$$\frac{\partial u}{\partial t} = \frac{9}{2}\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 0.9, \quad t > 0,$$

with the boundary conditions, $u(0,t) = 0$ and $u(0.9,t) = 1$
and the initial condition $u(x,0) = \sin \pi x$ for $0 < x < 0.9$.
By using explicit method, solve the heat equation up to second level $(t \leq 0.04)$ by taking $\Delta x = h = 0.3$ and $\Delta t = k = 0.02$.

(9 marks)

(b) A 6 cm by 5 cm rectangular silver plate has being heat uniformly generated at each point at the rate $q = 1.5\ \text{cal/cm}^3\text{s}$. Let x represent the distance along the edge of the plate of length 6 cm and y be the distance along the edge of the plate of length 5 cm. Suppose that the temperature u along the edges is kept as the following temperatures:

$$u(x,0) = x(6-x), \quad u(x,5) = 0 \quad 0 \leq x \leq 6$$
$$u(0,y) = y(5-y), \quad u(6,y) = 0 \quad 0 \leq y \leq 5$$

where the origin lies at a corner of a plate with coordinates $(0,0)$ and the edge lie along the positive x-axis and y-axis. The steady state temperature $u = u(x,y)$ satisfies Poisson's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{q}{K}, \quad 0 < x < 6, \quad 0 < y < 5,$$

where K, the thermal conductivity, is $1.04\ \text{cal/((cm)(degree)(s))}$. Use the finite difference method with $h = 1.5$ and $k = 2.5$ to approximate the temperature $u(x,y)$.

(16 marks)

Q2 Consider the heat flow equation, $\frac{d}{dx}\left(A(x)k(x)\frac{dT(x)}{dx}\right)+Q(x)=0$ for $1\leq x\leq 3$ on a fin consisting of three nodes and two elements, as shown in Figure **Q2.** In this equation

$T(x)$ is the temperature at length,
$A(x)$ is the cross-sectional area,
$k(x)$ is the thermal conductivity and
Q is the heat supply per unit time and per unit length.

Given the following values: $A(x)$ = 1 unit², $k(x)$ = 1 unit and Q = 1 unit. The boundary conditions are given as $T_1 = T|_{x=1} = 0°C$ and the flux, $q_3 = q|_{x=3} = 10$ unit. The shape functions are defined as below:

$$N_1(x)=\begin{cases} N_1^1(x)=-x+2, & \text{for } x \text{ in the element 1,} \\ 0, & \text{otherwise,} \end{cases}$$

$$N_2(x)=\begin{cases} N_2^1(x)=x-1, & \text{for } x \text{ in the element 1,} \\ N_2^2(x)=-x+3, & \text{for } x \text{ in the element 2,} \\ 0 & \text{otherwise,} \end{cases}$$

$$N_3(x)=\begin{cases} N_3^2(x)=x-2, & \text{for } x \text{ in the element 2,} \\ 0, & \text{otherwise.} \end{cases}$$

By using the Galerkin method for approximation on the linear model, $T=\alpha_1+\alpha_2 x$, find the temperature at each nodal point $T_2 = T|_{x=2}$ and $T_3 = T|_{x=3}$.

(25 marks)

SECTION B

Q3 (a) The manufacture of an automobile requires painting, drying and polishing. A motorcar company produces three type of cars: a sedan, a convertible and an SUV. The time requires (in hours) for each job for those cars are given in the table **Q3**. The last column shows the total time allocation (in hour) for those jobs in a certain week.

Table Q3

	Sedan	**Convertible**	**SUV**	**Total**
Painting	2	4	10	240
Drying	8	5	1	69
Polishing	4	16	1	41

(i) By assuming x as Sedan, y as Convertible and z as SUV, form the system of linear equations.

(ii) determine how many of each type of cars are produced by using Crout method.

(iii) Compare the results using Gauss-Seidel Iteration. Start with $\mathbf{X}^{(0)} = (\mathbf{6} \;\; \mathbf{0} \;\; \mathbf{23})^{\mathrm{T}}$.

(16 marks)

(b) The following simultaneous nonlinear equations $y = x^2 - 3x + 2$ and $y = e^{-x}$ are illustrated in Figure **Q3**.

(i) By Intermediate Value Theorem, show that is a value of x in the interval of $[0, 1.5]$ which is the intersection of the simultaneous nonlinear equations above.

(ii) By using Newton-Raphson method, find the x-value. Start with $x_0 = 0$.

(9 marks)

Q4 (a) Show that the following data construct the natural cubic spline below:

x	1	2	3
$f(x)$	1	1	0

$$S(x)=\begin{cases} -\frac{1}{4}(x-1)^3+(2-x)+\frac{5}{4}(x-1), & x\in[1,2] \\ -\frac{1}{4}(3-x)^3+\frac{5}{4}(3-x), & x\in[2,3] \end{cases}$$

(15 marks)

(b) By referring to the natural cubic spline in (a), complete the following table.

x	1.0	1.5	2.0	2.5	3.0
$f(x)$	1.0		1.0		0.0

Hence, find approximate values of $f'(2)$ with $h=0.5$. Use

(i) 2-point backward difference formula.
(ii) 3-point central difference formula.
(ii) 3-point forward difference formula.
(iii) 5-point difference formula.

(10 marks)

Q5 (a) Find the integral of $f(x)$ by using appropriate Simpson's rule between $x = 1.0$ until $x = 1.8$ for following data.

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.828	3.107

(6 marks)

(b) Given that $A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 6 & -2 \\ 0 & -1 & 3 \end{pmatrix}$ with three positive eigenvalues. By using the initial vector $\mathbf{v}^{(0)} = (0 \quad 1 \quad 0)^T$ and convergence criterion $|m_{k+1} - m_k| < \varepsilon = 0.005$.

(i) Compute the dominant eigenvalue and the corresponding eigenvector of the matrix A using Power method.

(ii) Use the Shifted Power method to compute the smallest eigenvalue and the corresponding eigenvector of the matrix A.

(12 marks)

(c) Solve the following ordinary differential equation

$$\frac{dy}{dx} + y = -2x, \; y(0) = -1,$$

with uniform step size $h = 0.1$ at interval [0, 0.2] by using fourth-order Runge-Kutta method.

(7 marks)

- END OF QUESTION -

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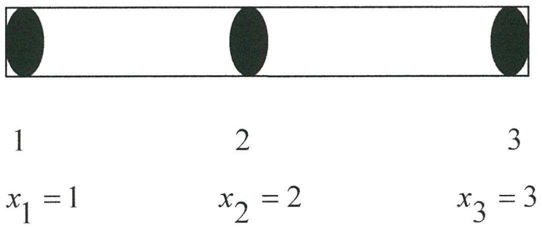

FIGURE Q2

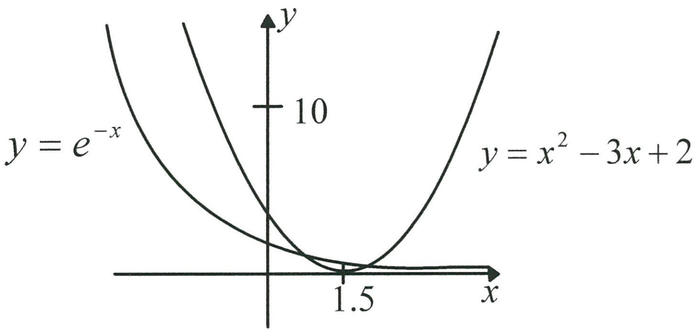

FIGURE Q3

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FORMULAS

Nonlinear equations

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}, \ i = 0,1,2,\ldots$

System of linear equations

Crout Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & \vdots \\ l_{31} & l_{32} & l_{33} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & 1 & u_{23} & \cdots & u_{2n} \\ 0 & 0 & 1 & \cdots & u_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 1 \end{pmatrix}$$

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, \ \forall i = 1, 2, 3, .., n.$

Interpolation

Natural Cubic Spline:

$$\left. \begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned} \right\}, \quad k = 0, 1, 2, 3, \ldots, n-1,$$

$b_k = 6(d_{k+1} - d_k), \ k = 0, 1, 2, 3, \ldots, n-2.$

$$m_0 = 0,$$

$$m_n = 0,$$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, \qquad k = 0, 1, 2, 3, \ldots, n-2.$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k).$$

$k = 0, 1, 2, 3, \ldots, n-1.$

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Numerical Differentiation

2-point forward difference: $f'(x) \approx \dfrac{f(x+h)-f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x)-f(x-h)}{h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h)-f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

Numerical Integration

Simpson $\frac{1}{3}$ Rule: $\displaystyle\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0+f_n+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2\\ i\text{ even}}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ rule : $\displaystyle\int_a^b f(x)\,dx \approx \frac{3}{8}h\left[\begin{array}{c}(f_0+f_n)+3(f_1+f_2+f_4+f_5+\cdots+f_{n-2}+f_{n-1})\\ +2(f_3+f_6+\cdots+f_{n-3})\end{array}\right]$

Eigen value

Power Method: $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0, 1, 2, \ldots..$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

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Partial Differential Equation
Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Finite Element Method

Heat flow problem in 1 dimension for $a \le x \le b$

$N(x) = [N_1(x) \ N_2(x) \ \cdots \ N_n(x)]$

$N_m(x) = [N_m^e(x) \ ]$ is global shaped function for element e at node m

$$T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}$$ is the temperature vector at node

$$\mathbf{K}T = \mathbf{f}_\text{b} - \mathbf{f}_\text{L}$$

where

stiffness matrix, $\mathbf{K} = \int_a^b \mathbf{B}^T Ak\mathbf{B} \, dx$ **or**

$$K_{ij} = \int_a^b \frac{dN_i}{dx} Ak \frac{dN_j}{dx} \, dx$$ is a square matrix with dimension $n \times n$,

boundary vector, $\mathbf{f}_\text{b} = -[\mathbf{N}^T Aq]_a^b$ have the dimension $n \times 1$,

load vector, $\mathbf{f}_\text{L} = -\int_a^b \mathbf{N}_i Q(x) \, dx$ have the dimension $n \times 1$.