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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS 1
COURSE CODE	:	BFC 13903
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2017 /JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE** QUESTIONS ONLY

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THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

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Q1 (a) Use the graph of the function $y = f(x)$ in **Figure Q1** to find the following limits:

(i) $\lim_{x \to 2^-} f(x)$

(ii) $\lim_{x \to 2^+} f(x)$

(iii) $\lim_{x \to 2} f(x)$

(iv) $\lim_{x \to -2^-} f(x)$

(v) $\lim_{x \to -2^+} f(x)$

(vi) $\lim_{x \to -2} f(x)$

(vii) $\lim_{x \to \infty} f(x)$

(viii) $\lim_{x \to -\infty} f(x)$

(4 marks)

(b) Evaluate each of the following limits by using L'Hopital's rule:

(i) $\lim_{x \to \infty} \frac{3x - 5}{6x + 8}$.

(3 marks)

(ii) $\lim_{x \to 1} \frac{\mathrm{x} + \cos(\pi x)}{x^2 - 1}$.

(3 marks)

(iii) $\lim_{x \to \infty} \frac{\ln x}{x^{\frac{1}{3}}}$.

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(4 marks)

(c) Determine the limits and examine whether the following functions are continuous at x = 1.

(i) $f(x) = \begin{cases} 4 - x^2 & if\ x < 1 \\ 2 + x^2 & if\ x > 1 \end{cases}$

(3 marks)

(ii) $f(x) = \begin{cases} 1 & if\ 0 \le x < 1 \\ x^2 & if\ x \ge 1 \end{cases}$

(3 marks)

Q2 (a) By using **Chain Rule**, differentiate $y = cot\left(\frac{\sin x}{x}\right)$.

(6 marks)

(b) Find dy/dx if $y^2 = x^2 + \sin xy$.

(8 marks)

(c) Find the slope of tangent line to $x = t^5 - 4t^3$, $y = t^2$ at the point $t = -2$.

(6 marks)

Q3 (a) Find the set of value of x for which the function

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 3x + 4$$

(i) is increasing.

(ii) is decreasing.

(4 marks)

(b) Find the set of value of x for which the function

$$f(x) = 4x^3 - 5x^2 - 10x + 11$$

(i) is concave upwards.

(ii) is concave downwards.

(4 marks)

(c) The given **Figure Q3** shows a combined solid which consists of a cuboid surmounted by a semi cylinder with a common face PQRS. The breadth and the length of the cuboid are x cm and $2x$ cm respectively and its height is y cm. Given that the total surface area of its combined solid is 8000 cm^2, prove that $y = \frac{1}{24x}[32000 - (8 + 5\pi)x^2]$.

(12 marks)

Q4 (a) Evaluate each of the followings:

(i) $\int_0^1 3(e^{-5t} + \sqrt{t})\, dt.$

(2 marks)

(ii) $\int_1^4 \left(\frac{1}{x} - x^2\right) dx.$

(2 marks)

(b) Integrate the following by using **substitution** method.

$$\int \frac{\cos 5x}{e^{\sin 5x}}\, dx.$$

(4 marks)

(c) Integrate the following by using **partial fraction** method.

$$\int \frac{x+1}{x(x^2+2)}\, dx.$$

(6 marks)

(d) Integrate the following by using **by parts** method.

$\int (\ln x)^2\, dx.$

(6 marks)

Q5 (a) Given $f(x) = 25x - \sin 2x$.

(i) Find the derivative of $f^{-1}(x)$ using **formula** $\frac{dy}{dx} = \frac{1}{dx/dy}$.

(5 marks)

(ii) Find the derivative of $f^{-1}(x)$ using **implicit differentiation.**

(5 marks)

(b) Differentiate $y = \sin^{-1}(x^3) + x\sin^{-1}x + \sqrt{1-x^2}$ with respect to x.

(7 marks)

(c) Integrate the following trigonometric function:

$$\int \frac{dx}{\sqrt{1-9x^2}}.$$

(3 marks)

Q6 (a) Find the area below $f(x) = -x^2 + 4x + 3$ and above $g(x) = -x^3 + 7x^2 - 10x + 5$ over the interval $1 \leq x \leq 2$.

(4 marks)

(b) Calculate the exact area of the surface obtained by rotating the curve $y = \sqrt{1 + e^x}$, at $0 \leq x \leq 1$.

(6 marks)

(c) Find the radius of curvature of $y = \sqrt{9 + 4x}$ at $x = 4$.

(10 marks)

- END OF QUESTION –

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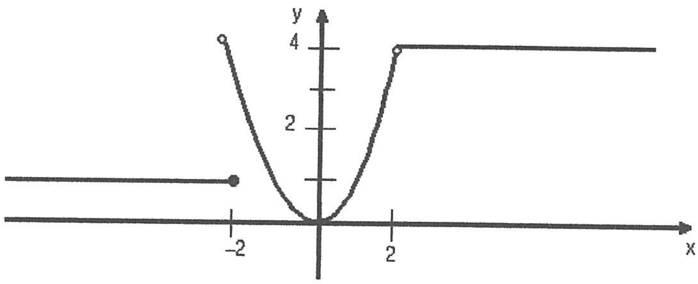

FIGURE Q1

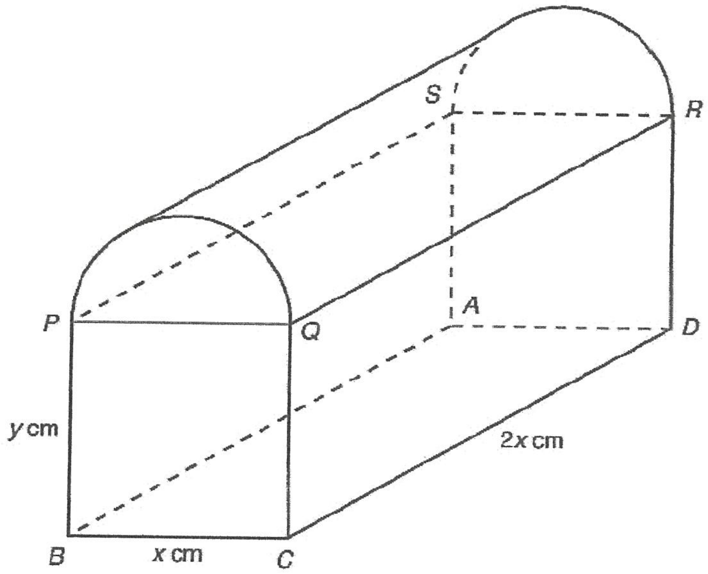

FIGURE Q3

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Formulae

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=-\text{sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=-\text{cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$

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<u>Formulae</u>

Trigonometric	Hiperbolic
$\cos^2 x+\sin^2 x=1$	$\sinh x=\dfrac{e^x-e^{-x}}{2}$
$1+\tan^2 x=\sec^2 x$	$\cosh x=\dfrac{e^x+e^{-x}}{2}$
$\cot^2 x+1=\operatorname{cosec}^2 x$	$\cosh^2 x-\sinh^2 x=1$
$\sin 2x=2\sin x\cos x$	$1-\tanh^2 x=\operatorname{sech}^2 x$
$\cos 2x=\cos^2 x-\sin^2 x$	$\coth^2 x-1=\operatorname{cosech}^2 x$
$\cos 2x=2\cos^2 x-1$	$\sinh 2x=2\sinh x\cosh x$
$\cos 2x=1-2\sin^2 x$	$\cosh 2x=\cosh^2 x+\sinh^2 x$
$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$	$\cosh 2x=2\cosh^2 x-1$
$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$	$\cosh 2x=1+2\sinh^2 x$
$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$	$\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$
$\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$	$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$
$2\sin x\cos y=\sin(x+y)+\sin(x-y)$	$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$
$2\sin x\sin y=-\cos(x+y)+\cos(x-y)$	$\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$
$2\cos x\cos y=\cos(x+y)+\cos(x-y)$	
Logarithm	**Inverse Hiperbolic**
$a^x=e^{x\ln a}$	$\sinh^{-1} x=\ln\left(x+\sqrt{x^2+1}\right)$, any x.
$\log_a x=\dfrac{\log_b x}{\log_b a}$	$\cosh^{-1} x=\ln\left(x+\sqrt{x^2-1}\right)$, $x\geq 1$
	$\tanh^{-1} x=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1<x<1$

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Formulae

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C.$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C.$
$\int \frac{dx}{\lvert a\rvert\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C.$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+C, \quad a>0.$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+C, \quad x>a$
$\int \frac{dx}{x^2-a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert>a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a.$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\text{cosech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a.$

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Formulae

Differentiation of Inverse Functions

y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$, $\lvert u\rvert < 1$.
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$, $\lvert u\rvert < 1$.
$\tan^{-1} u$	$\frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}} \cdot \frac{du}{dx}$, $\lvert u\rvert > 1$.
$\operatorname{cosec}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}} \cdot \frac{du}{dx}$, $\lvert u\rvert > 1$.
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}} \cdot \frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}} \cdot \frac{du}{dx}$, $\lvert u\rvert > 1$.
$\tanh^{-1} u$	$\frac{1}{1-u^2} \cdot \frac{du}{dx}$, $\lvert u\rvert < 1$.
$\coth^{-1} u$	$-\frac{1}{1-u^2} \cdot \frac{du}{dx}$, $\lvert u\rvert > 1$.
$\operatorname{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx}$, $0 < u < 1$.
$\operatorname{cosech}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{1+u^2}} \cdot \frac{du}{dx}$, $u \neq 0$.

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Formulae

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $K = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{K}$

Curvature of parametric curve

Curvature, $K = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{K}$

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