UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A** AND **THREE (3)** QUESTIONS IN **SECTION B**.

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THIS EXAMINATION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) Find the unit tangent vector, unit normal vector and curvature of the given vector-valued function.

$$\mathbf{r}(t) = \langle t, 3\cos t, 3\sin t \rangle$$

(10 marks)

(b) A force with the magnitude of 20 N acts directly upward from the xy-plane on an object with mass 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and its speed at time t.

(10 marks)

Q2 (a) Use Green's theorem to evaluate the following;

$$\oint_c (2y - x^2)dx + (4x + y^2)dy$$

where C is the boundary of the region in first quadrant bounded by $x^2 - 2x + y^2 = 0$ and $y = x$. In addition, evaluate the line integral directly.

(10 marks)

(b) Suppose C is the curve obtained by intersecting the plane $z = x$ and the cylinder $x^2 + y^2 = 1$, oriented counter-clockwise when viewed from above. Let S be the inside of this ellipse, oriented with the upward-pointing normal. If $\mathbf{F} = x\,\mathbf{i} + z\,\mathbf{j} + 2y\,\mathbf{k}$, verify Stokes' theorem by computing both $\oint_c F.dr$ and $\iint_S curl\, F.dS$.

(10 marks)

SECTION B

Q3 (a) For the function $f(x,y) = 7 - 4x - 3y$, sketch the level curve $f(x,y) = -7$, $f(x,y) = 0$, and $f(x,y) = 7$ in the domain of f in the plane.

(4 marks)

(b) The pressure P (kPa), volume V (L), and temperature T (K) of a mole of an ideal gas are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is 300 K and increasing at rate of 0.1 K/s and the volume is 100 L and increasing at rate of 0.2 L/s.

(10 marks)

(c) The period T of a pendulum is given by $T = 2\pi\sqrt{l/g}$, where l is the length of the pendulum and g is the acceleration due to gravity. Calculate the maximum percentage error made in computing g from this formula, if the percentage error in t and l are 0.5% and 1% respectively.

(6 marks)

Q4 (a) If $f(x,y) = x^2 - 4xy + y^3 + 4y$, find its critical points and determine whether $f(x,y)$ at that point is a local maximum, a local minimum, or a saddle point.

(13 marks)

(b) Find the domain and range for the function;

$$f(x,y) = \frac{xy}{x - 2y}\ .$$

(2 marks)

(c) Suppose the dimensions of a rectangular box are changed from 15, 13 and 11 to 15.03, 12.96 and 11.02 (in centimeter). Use total differential to approximate the changes in volume.

(5 marks)

Q5 (a) Calculate the iterated integral for the function $f(x,y) = 4x^3 + 6xy^2$ on the rectangle (1,3) and (-2,1).

(6 marks)

(b) Use a polar coordinates method to find the area enclosed by the three-petaled rose $r = \sin 3\theta$ as shown in **Figure Q5.**

(7 marks)

(c) Let G be the wedge in the first octant cut from the cylindrical solid $y^2 + z^2 \leq 1$ by the plane $y = x$ and $x = 0$. Calculate $\iiint_G z\, dV$.

(7 marks)

Q6 (a) Let R be the annular region lying between two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$. Sketch the region R and evaluate $\iint_R (x^2 + y)\, dA$.

(10 marks)

(b) Use triple integration in cylindrical coordinates to find the volume of the solid G that is bounded above by the hemisphere, $= \sqrt{36 - x^2 - y^2}$, below by the *xy*-plane, and laterally by the cylinder $x^2 + y^2 = 9$.

(10 marks)

- END OF QUESTIONS –

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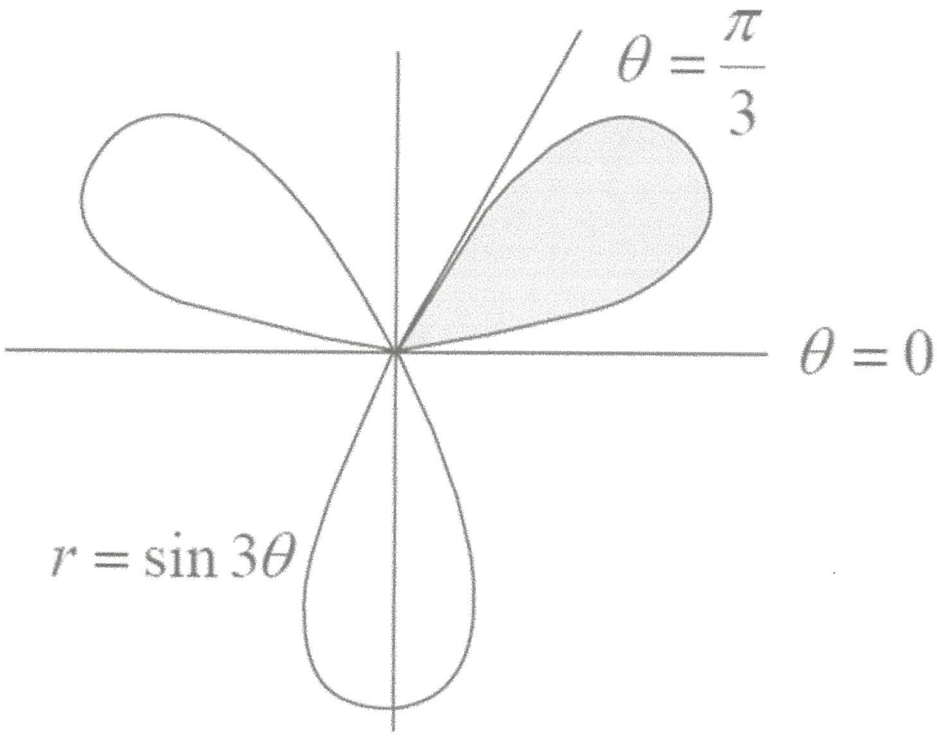

FIGURE Q5

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Formulae

Implicit Partial Differentiation:

$$\frac{\partial z}{\partial x} = -\frac{f_x(x,y,z)}{f_z(x,y,z)} \quad or \quad \frac{\partial z}{\partial y} = -\frac{f_y(x,y,z)}{f_z(x,y,z)}$$

Small Increment, Estimating Value:

Total differential/approximate change, $\partial z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$
Exact change, $dz = f(x_1, y_1) - f(x_0, y_0)$
Approximate value, $z = f(x_0, y_0) + dz$
Exact value, $z = f(x_1, y_1)$

Error: $|dz| = \left|\frac{\partial z}{\partial x}dx\right| + \left|\frac{\partial z}{\partial y}dy\right|$ and **Relative error:** $\left|\frac{dz}{z}\right| = \left|\frac{\partial z}{\partial x}\frac{dx}{z}\right| + \left|\frac{\partial z}{\partial y}\frac{dy}{z}\right|$

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\cos\theta\sin\phi$, $y = \rho\sin\theta\sin\phi$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$ and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

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Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

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Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

Gauss Theorem: $\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iiint_G \nabla \bullet \mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2\delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2\delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

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Centre of gravity, $(\bar{x},\bar{y},\bar{z})=\left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x=\iiint_G \left(y^2+z^2\right)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y=\iiint_G \left(x^2+z^2\right)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z=\iiint_G \left(x^2+y^2\right)\delta(x,y,z)\,dV$