UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	DISCRETE STRUCTURE
COURSE CODE	:	BIC 10103
PROGRAMME CODE	:	BIS / BIP / BIW / BIM
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) PLEASE WRITE YOUR ANSWERS IN THIS QUESTION BOOKLET C) CALCULATOR CANNOT BE USED

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

Q1 (a) Given that, $P(x)$: x is even, $Q(x)$: x is prime number and $R(x,y)$: $x + y$ is even. The variables of x and y represent integers. Write an English sentence for each of the following.

(i) $\forall x \, \exists y \, R(x,y)$

(2 marks)

(ii) $\neg \, (\exists x \, P(x))$

(2 marks)

(iii) $\forall x \, (\neg \, Q(x))$

(2 marks)

(b) State the converse, inverse, and contra-positive for each of the following implications:

i. Maria will find a good job when she learns discrete mathematics.

(3 marks)

ii. Being divisible by 3 is a necessary for this number to be divisible by 9.

(3 marks)

(c) Show that $(p \vee \sim q) \wedge (\sim p \vee \sim r \vee q) \wedge (p \vee \sim p)$ is not a tautology.

(8 marks)

Q2 (a) Let $A = \{1, 2, 4, 5, 7, 8\}$ and $B = \{x | (x \in Z^+) \wedge (x < 10)\}$. Write the element(s) for the following sets.

(i) $A \cup B$

(1 mark)

(ii) $A \cup \emptyset$

(1 mark)

(iii) $B - A$

(1 mark)

(b) Let A and B be the sets. The cross product of A and B is the set $A \times B = \{(a, b) | a \in A \wedge b \in B\}$.

Theorem: If $|A| = m$ and $|B| = n$ then $|A \times B| = m \times n$.

Use an example to demonstrate that the above theorem is true.

(3 marks)

(c) Let $U = \{d, i, s, c, r, e, t, e, m, a, t, h, e, m, a, t, i, c, s\}$ be the universal set. Let $S = \{x \in U \mid x \in \{s, e, c, r, e, t, s\}\}, T = \{x \in U \mid x \in \{t, h, e, m, e, s\}\}$ and $C = \{x \in U \mid x \in \{t, a, c, t, i, c, s\}\}$ be the subsets of the universal set.

(i) Draw a Venn diagram describing U, S, T and C.

(5 marks)

(ii) Write the elements for the following sets.

$S \cup T$ and $T - C$

(2 marks)

Q3 (a) Prove that $49^n + 16n - 1$ is divisible by 64.

(10 marks)

(a) Use mathematical induction to prove that

$$\sum_{i=1}^{n} i^4 = \frac{n(n+1)\,(2n+1)\,(3n^2+3n-1)}{30}$$

is true for all positive integers n.

(10 marks)

Q4 (a) Estimate worst case, best case and average case of algorithm in **Figure Q4(a).**

```
Algorithm:
      k ←1
      found← false
      while (k≤ n) and (not found) do
      if a_k = x then
                  found ←true
      else
                  k← k + 1
      endif
      endwhile
            { k > n or found }
      if found then { x found }
                  idx ←k
            else
            idx← 0 { x not found }
      end if
```

Figure Q4(a)

(8 marks)

(b) Create an algorithm with a single loop using $T(n) = 4n + 2$.

(8 marks)

Q5 (a) Find an Euler circuit from vertex A in **Figure Q5(a)**.

(9 marks)

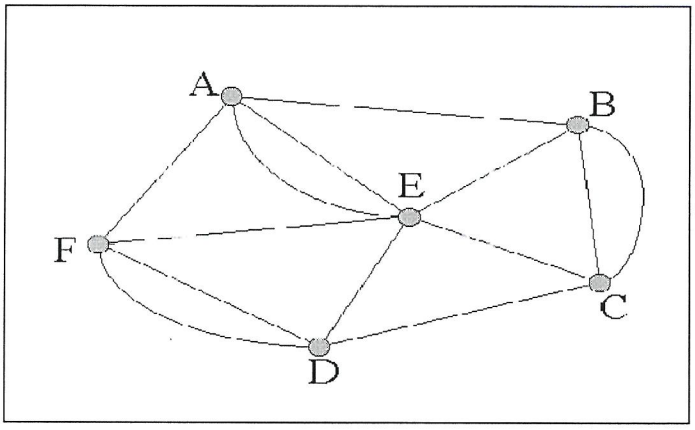

Figure Q5(a)

(b) Answer Q5(b) (i)-(ii), based on **Table 1**. A travelling salesman wants to visit 5 cities exactly once and return to his starting point. Suppose that the salesman wants to visit D, T, K, G, and S and its distances in miles are presented in **Table 1**.

Table 1 : Distances in miles for 5 cities

Cities/Miles	D	T	K	G	S
D	0	58	135	147	98
T	58	0	133	167	142
K	135	133	0	56	137
G	147	167	56	0	113
S	98	142	137	113	0

(i) Draw a connected graph between the 5 cities.

(5 marks)

(ii) Find the minimum total distance in his travelling based on *Hamiltonian* circuit approach. List at least 12 different circuits in your answer.

(7 marks)

Q6 (a) Identify two ways to represent a relation.

(2 marks)

(b) Let $A = [1, 2, 3, 4, 5\}$ and let R be the relation on A defined as follows:
$R = \{(1, 3), (1, 4), (2, 1), (2, 2), (2, 4), (3, 5), (5, 2), (5, 5)\}$

(i) Write the matrix representation for R.

(5 marks)

(ii) Draw the graphical representation for R.

(3 marks)

- END OF QUESTION -