CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	: DIGITAL CONTROL
COURSE CODE	: BER 4113/BEM 4713
PROGRAMME	: BEE
EXAMINATION DATE	: APRIL/MAY 2011
DURATION	: 2 HOURS 30 MINUTES
INSTRUCTION	: ANSWER **FOUR (4)** QUESTIONS ONLY

THIS PAPER CONSISTS OF NINE (9) PAGES

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Q1 (a) A unity feedback closed loop system plant with the digital controller is shown in Figure Q1. Design a digital controller with an inter-sampling interval of 0.5 s to deadbeat control a plant having the transfer function

$$G_p(s) = \frac{2}{s(s+2)}$$

(13 marks)

(b) Assuming that the input signal is a unit step, obtain and draw the output response of the system in Q1 (a).

(12 marks)

Q2 (a) A system with the characteristic equation $P(z) = 0$ is given by

$$P(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n$$

where $a_0 > 0$.

A general form of the Jury Stability Table is given in Table Q2. According to the Jury Stability Test, what are the criterions to be fulfilled in order to make the system stable?

(7 marks)

(b) A digital control system has a following characteristic equation:

$$P(z) = z^4 - 1.2z^3 + 0.07z^2 + 0.3z - 0.08 = 0$$

By using the Jury Stability Test, examine the stability of the system.

(18 marks)

Q3 (a) Using the complex translation theorem, obtain the z transforms of:

$$y(t) = e^{-at} e^{-at} sinwt$$

(**Hint**: Complex translation theorem - If $x(t)$ has the z transform $X(z)$, then the z transform of $e^{-at}x(t)$ can be given by $X(ze^{at})$).

(6 marks)

(b) A system with Fibonacci series is given by the difference equation:

$$x(k+2) = x(k+1) + x(k)$$

where $x(0) = 0$ and $x(1) = 1$.

If the input to the system, is the Kronecker delta function $\delta_0(kT)$, where

$$\delta_0(kT) = 1 \quad \text{for } k = 0$$
$$= 0 \quad \text{for } k \neq 0,$$

write a MATLAB program to generate the Fibonacci series up to $k = 100$.

(9 marks)

(c) Figure Q3(c) shows the discrete-time control system. Proof that the z-transform of the output of the system is given by:

$$C(z) = \frac{\overline{GR}(z)}{1 + \overline{GH}(z)}.$$

(10 marks)

Q4 Figure Q4 shows the block diagram representation of a discrete control system. If the sampling period (T) is 0.1s and the input is a unit step function,

a) Determine the open loop pulse transfer function.

(6 marks)

b) Determine the closed loop pulse transfer function.

(3 marks)

c) Draw the root locus for the system (use graph scale, 1 unit : 5 cm)

(16 marks)

Q5 (a) Obtain the state equation and the output equation in matrix form for an armature-controlled direct current motor as shown in Figure Q5. It is assumed that the armature inductance is very small and can be neglected. Assume that the state variables are given by

$$x_1(t) = \theta_m(t)$$
$$x_2(t) = \omega_m(t)$$

the output $y(t) = \theta_m(t) = x_1(t)$ and the input $u(t) = v_a(t)$. The parameters for the motors are : J_m – moment of inertia of the motor, B_m – viscous frictional constant of the motor, k_b – back emf constant and k_t – motor torque constant.

(15 marks)

(b) Explain clearly with the help of relevant diagram, how you would implement state variable feedback digital control into the system in Figure Q5. State any components that you may require.

(10 marks)

Q6 The discrete-time state equation of a control system is given by:

$$\underline{x}(k+1) = G\underline{x}(k) + H\underline{u}(k)$$

where G and H are (n x n) and (n x r) matrices respectively.

(a) State the condition where state variable feedback can be applied to this system and give a method to test the condition.

(5 marks)

Given that

$$G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.16 & 0.84 & 0 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(b) Show that state variable feedback can be applied to this system with the given matrices G and H.

(6 marks)

(c) Design a suitable a suitable feedback gain matrix K if the control signal is given by u(k) = -K$\underline{x}$(k), the response of the closed-loop system is deadbeat.

(14 marks)

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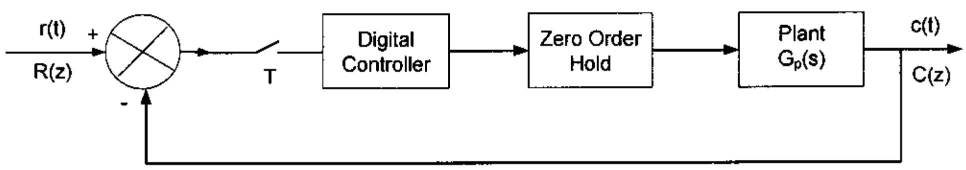

Figure Q1

Table Q2

Row	z^0	z^1	z^2	z^3	. . .	z^{n-2}	z^{n-1}	z^n
1	a_n	a_{n-1}	a_{n-2}	a_{n-3}	. . .	a_2	a_1	a_0
2	a_0	a_1	a_2	a_3	. . .	a_{n-2}	a_{n-1}	a_n
3	b_{n-1}	b_{n-2}	b_{n-3}	b_{n-4}	. . .	b_1	b_0	
4	b_0	b_1	b_2	b_3	. . .	b_{n-2}	b_{n-1}	
5	c_{n-2}	c_{n-3}	c_{n-4}	c_{n-5}	. . .	c_0		
6	c_0	c_1	c_2	c_3	. . .	c_{n-2}		
. . .	. . .							
$2n-5$	p_3	p_2	p_1	p_0				
$2n-4$	p_0	p_1	p_2	p_3				
$2n-3$	q_2	q_1	q_0					

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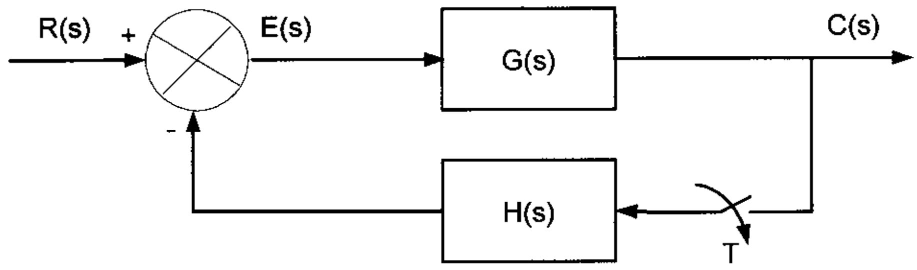

Figure Q3(c)

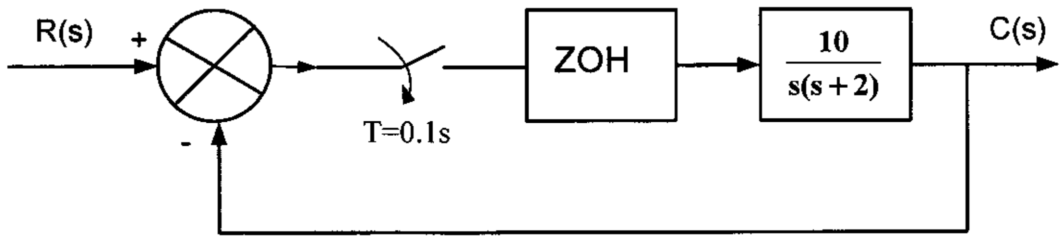

Figure Q4

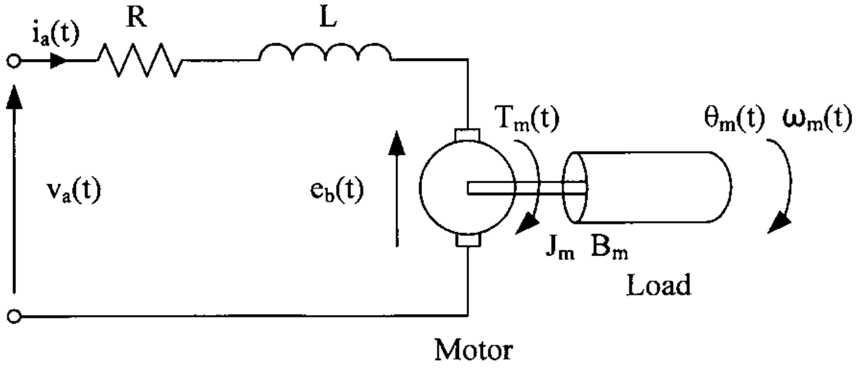

Figure Q5

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Table 1: Table of z Transform

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1, $k = 0$ 0, $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1, $n = k$ 0, $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

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	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T + e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T + e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			$k^2 a^{k-1}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			$k^3 a^{k-1}$	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			$k^4 a^{k-1}$	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

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Table 2: z Transform of $x(k+m)$ and $x(k-m)$

Discrete function	z Transform
$x(k+4)$	$z^4X(z) - z^4x(0) - z^3x(1) - z^2x(2) - zx(3)$
$x(k+3)$	$z^3X(z) - z^3x(0) - z^2x(1) - zx(2)$
$x(k+2)$	$z^2X(z) - z^2x(0) - zx(1)$
$x(k+1)$	$zX(z) - zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$