UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2010/2011

COURSE NAME	:	DIGITAL CONTROL
COURSE CODE	:	BER 4113/BEM 4713
PROGRAMME	:	4BEE
EXAMINATION DATE	:	NOVEMBER/DECEMBER 2010
DURATION	:	2 ½ HOURS
INSTRUCTION	:	ANSWER FOUR (4) QUESTIONS ONLY

THIS PAPER CONSISTS OF NINE (9) PAGES

Q1 (a) A function $f(t) = 4\sin 6t$ is sampled every T = 0.1 s. Find the z transform of the resultant number sequence.

(4 marks)

(b) A function $f(t) = A\, cos\omega t$ is sampled every T = 0.2 s. The z transform of the resultant number sequence is

$$F(z) = \frac{3z(z - 0.6967)}{z^2 - 1.393zz + 1}$$

Solve for A and ω.

(9 marks)

(c) Figure Q1(c) shows the discrete-time control system. Determine the Closed Loop Pulse Transfer Function of the system, $C(z)/R(z)$ in terms of G_1, G_2 and H.

(12 marks)

Q2 Figure Q2 show a block diagram of a control system with plant G(s) and digital compensator D(z). The Plant function is given by

$$G(s) = \frac{1 - e^{-Ts}}{s(s+1)}$$

and the digital compensator is a PI compensator which is given by

$$D(z) = \frac{K_I z}{z - 1} + K_p$$

The design specification for this system requires that the steady state error to a unit ramp input must be less than 0.01.

(a) Determine the velocity error constant if the sampling period T = 0.1s and K_I =10.

(10 marks)

(b) Using the Jury Stability Test, determine the range of K_p in order to fulfilled the design specification mentioned.

(15 marks)

Hint:

$$K_v = \lim_{z\to 1}\frac{1}{T}(z - 1)D(z)G(z)$$

Q3 Design a digital controller with an inter-sampling interval of 1 s to deadbeat control a plant having the transfer function 1/[s(s + 1)]. Assume that the input is a unit step signal. Obtain and draw the response of this system.

(25 marks)

Q4 A digital control system with a unity negative feedback has the following details:

Digital controller gain = K,
Pulse transfer function of the plant preceded by a zero-order hold

$$G(z) = \frac{0.3679z + 0.1859}{z^2 - 1.2308z + 0.2308}$$

(a) Draw the block diagram of this system.

(2 marks)

(b) Draw the root locus diagram for this system. (Graph scale: 1 unit : 5 cm)

(15 marks)

(c) Determine the value of the critical gain.

(3 marks)

(d) Determine the gain K such the number of samples per cycle of damped sinusoidal oscillation is 10. Give all the closed-loop poles with this gain K.

(5 marks)

Q5 (a) Obtain the state equation and the output equation in matrix form for an armature-controlled d.c. motor with non zero armature inductance as shown in Figure Q5. Assume that the state variables are given by

$$x_1(t) = i_a(t)$$
$$x_2(t) = \theta_m(t)$$
$$x_3(t) = \omega_m(t)$$

the output $y(t) = \theta_m(t) = x_2(t)$ and the input $u(t) = v_a(t)$. The parameters for the motors are : J_m – moment of inertia of the motor, B_m – viscous frictional constant of the motor, k_b – back emf constant and k_t – motor torque constant.

(15 marks)

(b) Explain clearly with the help of relevant diagram, how you would apply state variable feedback digital control into the system in Figure Q5. State any components that you may require.

(10 marks)

Q6 The state equation of a discrete-time control system is given by:

$$\underline{x}(k+1) = G\underline{x}(k) + Hu(k)$$

where

$$G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.5 & -0.2 & 1.1 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(a) Show that the system is controllable

(8 marks)

(b) Design a state variable feedback control system that will place the closed-loop eigenvalues at $z_1 = 0.5$, $z_2 = 0.5 + j0.5$ and $z_3 = 0.5 - j0.5$.

(17 marks)

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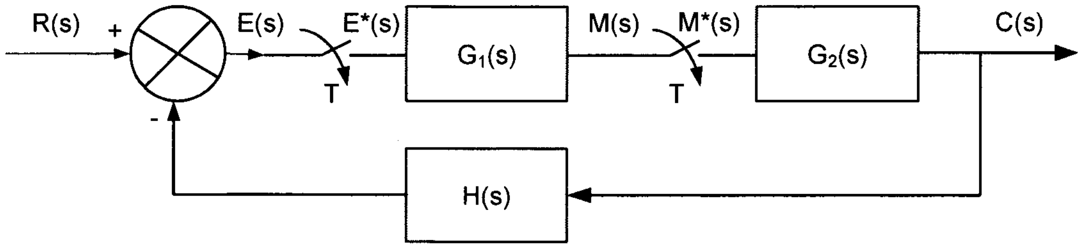

Figure Q1(c)

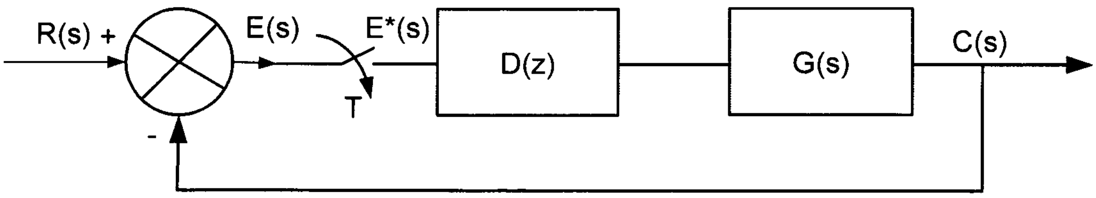

Figure Q2

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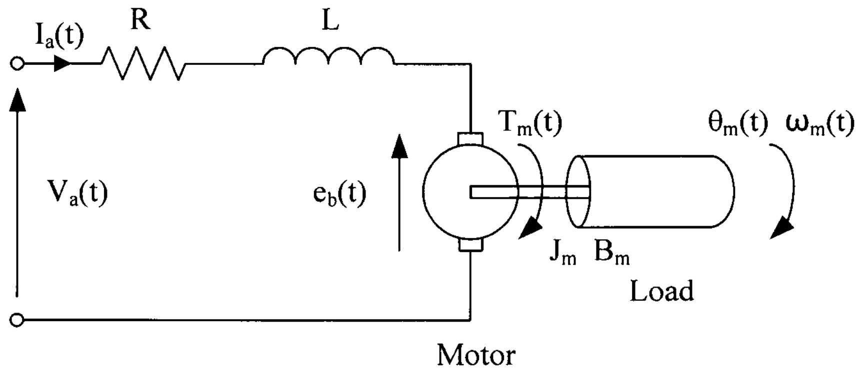

Figure Q5

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Table 1: Table of z Transform

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ $1, \quad k = 0$ $0, \quad k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ $1, \quad n = k$ $0, \quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

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	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT}) + (1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1 - 2z^{-1}\cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1 - z^{-1}\cos \omega T}{1 - 2z^{-1}\cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1 - 2e^{-aT}z^{-1}\cos \omega T + e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1 - e^{-aT}z^{-1}\cos \omega T}{1 - 2e^{-aT}z^{-1}\cos \omega T + e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			$k^2 a^{k-1}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			$k^3 a^{k-1}$	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			$k^4 a^{k-1}$	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

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Table 2: z Transform of $x(k+m)$ and $x(k-m)$

Discrete function	z Transform
$x(k+4)$	$z^4 X(z) - z^4 x(0) - z^3 x(1) - z^2 x(2) - zx(3)$
$x(k+3)$	$z^3 X(z) - z^3 x(0) - z^2 x(1) - zx(2)$
$x(k+2)$	$z^2 X(z) - z^2 x(0) - zx(1)$
$x(k+1)$	$zX(z) - zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1} X(z)$
$x(k-2)$	$z^{-2} X(z)$
$x(k-3)$	$z^{-3} X(z)$
$x(k-4)$	$z^{-4} X(z)$