UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEB 30503 / BEX 31803 / BEE 3213
PROGRAMME	:	BEB/BEC/BED/BEE/BEF/BEH/BEU
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **TWO (2)** QUESTIONS IN **SECTION A** AND **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

SECTION A

Q1. Given a signal,

$$x(n) = \begin{cases} 2^{n-1} & ; -2 \le n < 1 \\ 3^{-n} & ; \ 1 \le n < 3 \\ 0 & ; \textit{elsewhere} \end{cases}$$

(a) Determine the numeric sequences of the following signals.

(i) $M(n) = x(-n + 0.25) + u(n+1) - u(n)$

(ii) $N(n) = 2x(n) + r(n) - r(n-1) - u(n-3)$

(13 marks)

(b) A signal from a sensor has been measured as shown in Figure Q1. Calculate the energy of the even part of $w(n)$.

(7 marks)

Q2 (a) A Finite Impulse Response (FIR) filter has an impulse response and input signal given by $h(n) = \delta(n) + 2\delta(n-2) + 2\delta(n-3)$ and $x(n) = 2tri\left(\frac{n}{2}\right)$, respectively. Determine its response $y(n)$ by using the sum-by-column method.

(5 marks)

(b) Briefly explain the process of periodic convolution by using the cyclic method. Then, determine the output of the system by using this method, if the system input is $x(n) = \{\overset{\Downarrow}{1}, -2, 3\}$ and the impulse response is $h(n) = \{-2, \overset{\Downarrow}{1}, 0\}$.

(7 marks)

(c) Compute the cross-correlation of $r_{xh}(n)$ and $r_{hx}(n)$ for $x(n) = \{2, \overset{\Downarrow}{1}, 1, 2\}$ and $h(n) = \{4, \overset{\Downarrow}{2}, 3\}$. Find the relationship between $r_{xh}(n)$ and $r_{hx}(n)$.

(8 marks)

SECTION B

Q3 A group of students are asked to digitalize an analog voltage $x(t) = 10\sin(20\pi t) + 3\cos(2\pi t)\ V$ to FIVE (5) encoded samples using a system "A". Below are the requirements to complete the task.

TASK REQUIREMENT FOR SYSTEM "A"
The analog signal is sampled at $f_s = 10\ Hz$ and it will be quantized using **TWO (2)** bits system with a dynamic range of $\pm 4V$.

(a) Name the system "A" based on the information above

(1 mark)

(b) Illustrate the basic block diagram of system "A"

(2 marks)

(c) Determine the system Quantization Signal-to-Noise ratio (SNR_Q) in dB and show all the steps involved. Please note that the system uses uniform quantization process (rounding technique).

(17 marks)

Q4 (a) A discrete signal is given as $x(n) = 2\delta(n) - 2\delta(n-2)$. Calculate the Discrete Fourier Transform (DFT) of $x(n)$.

(4 marks)

(b) An input signal of a filter is written as

$$r(n) = \{\, 1, 2, 1 \,\} \Leftrightarrow R_{DFT}(k) = \{\, 4, -\frac{1}{2}, -j\sqrt{\frac{3}{4}}, A \,\}$$

Determine

i) The value of A

ii) $Y_{DFT}(k)$ if $y(n) = r(-n)$

iii) $W_{DFT}(k)$ if $w(n) = r^*(n)$

(3 marks)

(c) The Discrete Fourier Transform (DFT) of a discrete signal, $c(n)$ is known as $C_{DFT}(k) = \{8, -2j, 0, 2j\}$.

i) Illustrate the butterfly structure of Decimation in Time (DIT) Fast Fourier Transform Algorithm (FFT) technique.

ii) Determine discrete signal, c[n] based on your answer in **Q4(c)**

(13 marks)

Q5 (a) A signal, $x(n) = 2^n u(n+1)$ is used as input to the filter shown in Figure Q5. Analyze the output of the filter, $Y(z)$.

(5 marks)

(b) A lowpass filter with transfer function $H(z) = \dfrac{0.3249(z+1)}{1.3249z - 0.6751}$ operates at S = 10 *kHz*, and its cutoff frequency is f_c = 1 *kHz*. Use this filter to design a lowpass filter with a cutoff frequency of 3 *kHz*.

(7 marks)

(c) Design a lowpass filter with a cutoff frequency of 1 *kHz*. The sampling frequency is 10 *kHz*.

(8 marks)

Q6 (a) Based on the difference equation below, analyze the stability of the system.

$$y(n) - 3y(n-1) + 2y(n-2) - 2x(n-2) = 0$$

(5 marks)

(b) Calculate the z-transform and its region of convergence for the following discrete signal:

i) $x(n) = \{1, 5, \underset{\Uparrow}{3}, 11\}$

ii) $c(n) = (n-1)(2)^{n+2}u(n)$

(5 marks)

(c) Design an FIR highpass filter with cutoff frequency f_c = 2 *kHz* using sampling frequency S = 10 *kHz*. Use the Barlett window with $N = 9$.

(10 marks)

- END OF QUESTION -

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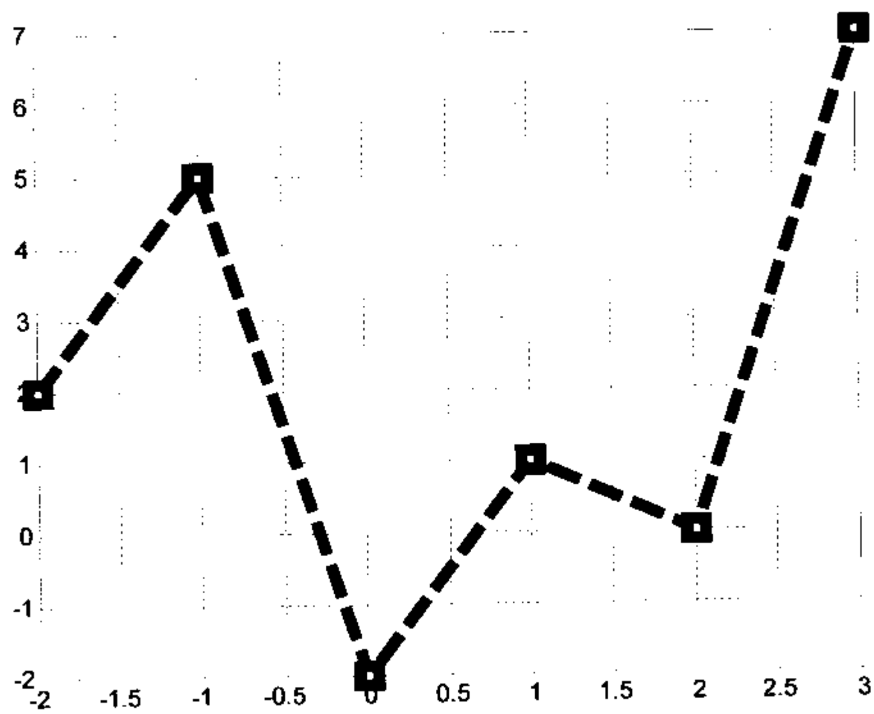

FIGURE Q1

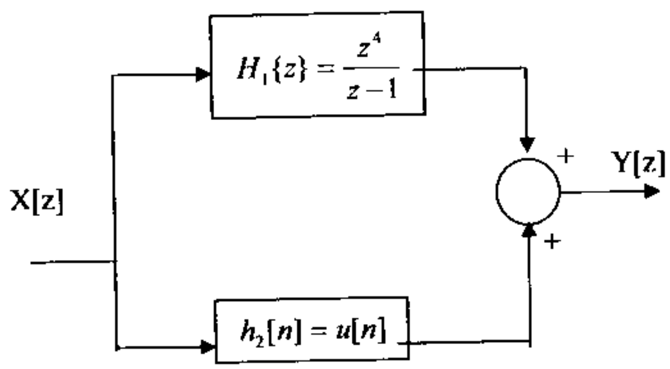

FIGURE Q5

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Table 1 : Properties of the *N*-sample DFT.

Signal x(n)	DFT X(k)
$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(k)+a_2X_2(k)$
$x[n-n_o]$	$X_{DFT}[k]e^{-\frac{j2\pi k n_o}{N}}$
$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$
$x[n]e^{j\frac{2\pi n k_0}{N}}$	$X_{DFT}[k-k_o]$
$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$
$x[-n]$	$X_{DFT}[-k]$
$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$
$x[n]\otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$
$x[n]\otimes\otimes y[n]$	$X_{DFT}[k]Y^*_{DFT}[k]$
$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$ $\quad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$	
$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even) $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n X_{DFT}[k]$ (N even)	
$\sum_{k=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$	

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Table 2 : A Short Table of z-transform Pairs.

Signal	z-transform
$\delta[n]$	1
$\delta[n-n_0]$	z^{-n_0}
$u[n]-u[n-N]$	$\frac{1-z^{-N}}{1-z^{-1}}$
$u[n]$	$\frac{z}{z-1}$
$\alpha^n u[n]$	$\frac{z}{z-\alpha}$
$(-\alpha)^n u[n]$	$\frac{z}{z+\alpha}$
$nu[n]$	$\frac{z}{(z-1)^2}$
$n\alpha^n u[n]$	$\frac{z\alpha}{(z-\alpha)^2}$

Table 3 : Properties of the z-Transform.

Signal	z-Transform
$x[n-N]$	$z^{-N}X(z)$
$x[-n]$	$X\left(\frac{1}{z}\right)$
$x[-n]u[-n-1]$	$X\left(\frac{1}{z}\right)-x[0]$ (for causal $x[n]$)
$\alpha^n x[n]$	$X\left(\frac{z}{\alpha}\right)$
$nx[n]$	$-z\frac{dX(z)}{dz}$
$\cos(n\Omega)x[n]$	$0.5[X(ze^{j\Omega})+X(ze^{-j\Omega})]$
$\sin(n\Omega)x[n]$	$j0.5[X(ze^{j\Omega})-X(ze^{-j\Omega})]$
$x[n]*h[n]$	$X(z)H(z)$

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Table 4 : Some Windows for FIR Filter Design.

Note: $I_0(x)$ is the modified Bessel Function of order zero.

Window	Expression $\omega[n]$, $-0.5(N-1) \le n \le 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right)$, $L > 0$
Bartlett	$1 - \frac{2\lvert n\rvert}{N-1}$
von Hann (Hanning)	$0.5 + 0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54 + 0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42 + 0.5\cos\left(\frac{2n\pi}{N-1}\right) + 0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4[n/(N-1)]^2}\right)}{I_0(\pi\beta)}$

Table 5 : Characteristics of the windowed spectrum for various windows.

Window	Peak Ripple $\delta_p = \delta_s$	Passband Attenuation A_{WP} (dB)	Peak Sidelobe Attenuation A_{WS} (dB)	Transition Width $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

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Table 6 : Digital-to-digital frequency transformations.

Form	**Band Edges**	**Mapping $s \rightarrow$**	**Parameters**
Lowpass to lowpass	Ω_C	$\dfrac{z-\alpha}{1-\alpha z}$	$\alpha = \dfrac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\dfrac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \dfrac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\dfrac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \dfrac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha K}{K+1}$, $A_2 = \dfrac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\dfrac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha}{K+1}$, $A_2 = \dfrac{1-K}{1+K}$

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Table 7 : Direct analog-to-digital transformations for bilinear design.

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

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Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=0}^{n}\alpha = \frac{n(n+1)}{2}$$

$$\sum_{k=0}^{n}\alpha^{2} = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=0}^{n}\alpha^{3} = \frac{n^{2}(n+1)^{2}}{4}$$

$$\sum_{k=0}^{n}\alpha^{k} = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n}k\alpha^{k} = \frac{\alpha\left[1-(n+1)\alpha^{n} + n\alpha^{n+1}\right]}{(1-\alpha)^{2}}$$

$$\sum_{k=0}^{n}k^{2}\alpha^{k} = \frac{\alpha\left[(1+\alpha)-(n+1)^{2}\alpha^{n} + (2n^{2}+2n-1)\alpha^{n+1} - n^{2}\alpha^{n+2}\right]}{(1-\alpha)^{3}}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty}\alpha^{k} = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty}\alpha^{k} = \frac{\alpha}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty}k\alpha^{k} = \frac{\alpha}{(1-\alpha)^{2}}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty}k^{2}\alpha^{k} = \frac{\alpha^{2}+\alpha}{(1-\alpha)^{3}}, \qquad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty}e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha > 0$$