



UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013**

COURSE NAME : ELECTROMAGNETIC
ENGINEERING

COURSE CODE : BEF 22903

PROGRAMME : 2 BEF

EXAMINATION DATE : JUNE 2013

DURATION : 3 HOURS

INSTRUCTION : ANSWER **FIVE (5)** QUESTIONS
ONLY

THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

- Q1** (a) Electrostatic concept is used in many areas of application. Describe one (1) of the applications of electrostatic in computer related industry. (4 marks)
- (b) A uniform line charge of density $\rho_l = 20 \text{ nC/m}$ exists at $x = 2\text{m}$, $y = -4\text{m}$. If two uniform sheets of charge with charge density $\rho_s = 10 \text{ nC/m}^2$ and $\rho_s = 30 \text{ nC/m}^2$ lie in the planes $z = -1 \text{ m}$ and $y = 2 \text{ m}$ respectively, calculate the electric field intensity, \vec{E} at point $(-2, -1, 4)$ due to the three charges distribution. (12 marks)
- (c) A spherical shell centered at the origin extends between $R = 2 \text{ cm}$ and $R = 3 \text{ cm}$. If the volume charge density is given by $\rho_v = 3R \times 10^{-4} \text{ C/m}^3$, Determine the total charge contained in the spherical shell. (4 marks)

- Q2** (a) The Gauss's law for static electric field is given by $\oint_S \vec{D} \cdot d\vec{s} = Q_{enc}$ where \vec{D} is the electric flux density and Q_{enc} is the net charge in the closed surface, S . Use Gauss's law to prove that the electric field in a perfect conductor cannot exist and this finding can be applied to build a facility to protect sensitive electronic devices. (6 marks)
- (b) A spherical charge distribution is given by

$$\rho_v = \begin{cases} \frac{15}{r^2} \text{ mC/m}^3, & 0 < r < 4 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Calculate the net flux crossing surface $r = 3 \text{ m}$ and $r = 7 \text{ m}$. (5 marks)
- (ii) Determine \vec{D} at $r = 2\text{m}$ and $r = 5\text{m}$. (5 marks)

- (c) Consider a thin spherical shell of radius a carries a uniform surface charge density, ρ_s . Use Gauss's law to determine the electric field intensity, \vec{E} at everywhere.

(4 marks)

- Q3** (a) The Biot Savart's law enables us to write the general results for the magnetic field due to an arbitrary current distribution. It is an experimental law predicted by Biot and Savart dealing with magnetic field strength at a point due to a small current element. Like Coulomb's law, Biot-Savart's law is the general law of magnetostatic field.

- (i) Define the Biot-Savart's law.

(3 marks)

- (ii) Discuss the similarities and differences between electric field and magnetic field.

(3 marks)

- (b) Consider a circular loop of radius a lying on x-y plane with a current I in the $+\hat{\phi}$ direction as shown in Figure Q3(b). Show that the magnetic field intensity, \vec{H} at point $P(0, 0, h)$ is given by

$$\vec{H} = \frac{Ia^2}{2(a^2 + z^2)^{3/2}} \hat{z} \quad (\text{A/m})$$

(7 marks)

- (c) Two parallel circular loops carrying a current of 40 A each are arranged as shown in Figure Q3(c). The first loop is situated in the x-y plane with its center at the origin, and the second loop's center is at $z = 2$. If the two loops have the same radius $a = 3$, determine the magnetic field intensity, \vec{H} at point $P(0, 0, 1)$.

(7 marks)

- Q4** (a) Ampere's law is an alternative formulation to obtain magnetic field intensity, \vec{H} and magnetic flux density, \vec{B} by the relation with current.

- (i) Define Ampere's circuital law.

(3 marks)

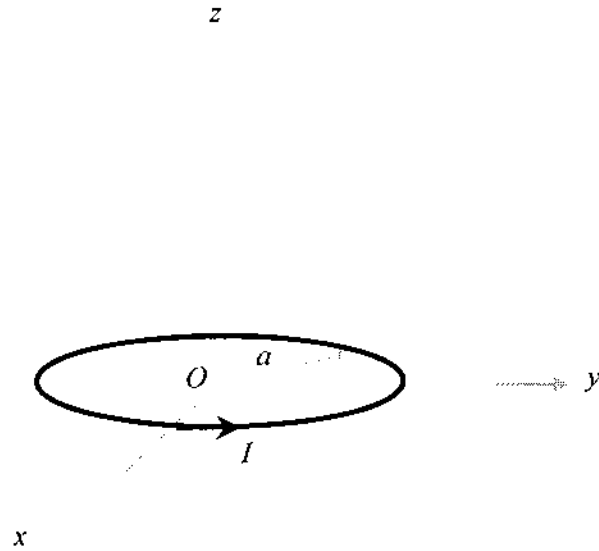
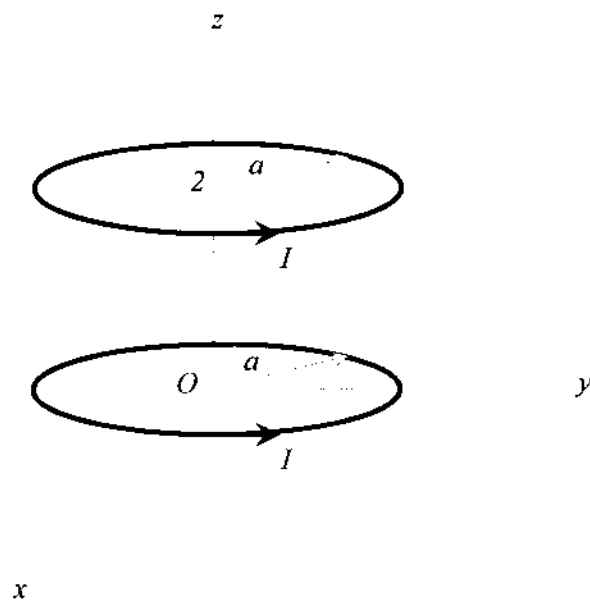
- (ii) By using Ampere's circuital law, calculate the magnetic field intensity, \vec{H} at point $P(4, 0, 0)$ caused by an infinitely long filamentary current, I along the y -axis as shown in Figure Q4 (a). (4 marks)
- (b) Consider an infinite long wire conductor carries current 6 mA in positive \hat{y} -direction is enclosed symmetrically by a cylindrical shell as shown in Figure Q4(b). The cylindrical shell has inner radius, $a = 3$ cm and outer radius, $b = 6$ cm and carries return current 4 mA in negative \hat{y} -direction.
- (i) Sketch the Amperian path at $r < a$, $a \leq r \leq b$ and $r > b$. (2 marks)
- (ii) Calculate magnetic field intensity \vec{H} at $r < a$, $a \leq r \leq b$ and $r > b$. (8 marks)
- (iii) Plot the magnitude of the magnetic field intensity H against distance R from the center of the cylinder. Interpret the results. (3 marks)
- Q5** (a) State Maxwell's equations both in differential and integral form related to static electric and magnetic fields. (4 marks)
- (b) A rectangular loop as shown in Figure Q5 (b) lies in the x - y plane at $z = 0$. Find the total force exerted on the rectangular loop located in free space:
- (i) If the magnetic flux density, \vec{B} is given by $\vec{B} = x\hat{x} + 2y\hat{y} + 3z\hat{z}$ T. (6 marks)
- (ii) If the magnetic flux density, \vec{B} is due to an infinitely long filamentary wire carrying current of 5 mA as shown in Figure Q5 (b) (ii). (10 marks)
- Q6** (a) Faraday's law states that the induced electromotive force (emf), V_{emf} in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit. With an aid of diagram, propose an experiment to prove the Faraday's Law. (6 marks)

- (b) A sliding bar as shown in Figure Q6 (b) is located at $x = 10t + 4t^3$, and the separation between two rails is 40 cm. If the magnetic flux density, $\vec{B} = 0.8x^2 \hat{z}$ Tesla, find the voltmeter reading at $t = 0.8$ s. (10 marks)
- (c) Consider a conductor joining the two ends of a resistor as shown in Figure Q6 (c). The time varying magnetic field is given by $\vec{B} = 0.4 \cos(120\pi t)$ Tesla. Assume that the magnetic field produced by $I(t)$ is negligible. Calculate the induced electromotive force, $V_{ab}(t)$ in the circuit. (4 marks)
- Q7** (a) The propagation of a plane wave has differences characteristic depend the medium used. Elaborate the plane wave propagation characteristic for:
- (i) Free space, (3 marks)
 - (ii) Lossless dielectric, (3 marks)
 - (iii) Good conductor. (3 marks)
- (b) The electric field in free space is given by
- $$\vec{E} = 50 \cos(10^8 t + \beta x) \hat{z} \quad \text{V/m}$$
- (i) Find the direction of wave propagation. (3 marks)
 - (ii) Calculate the phase constant, β and the time it takes to travel a distance of $\lambda/2$. (5 marks)
 - (iii) Sketch the wave at $t = 0$, $t = T/4$ and $t = T/2$. (3 marks)

-END OF QUESTION-

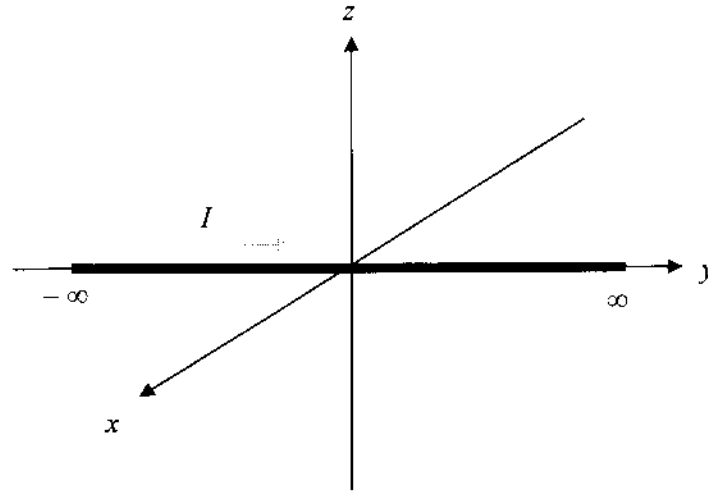
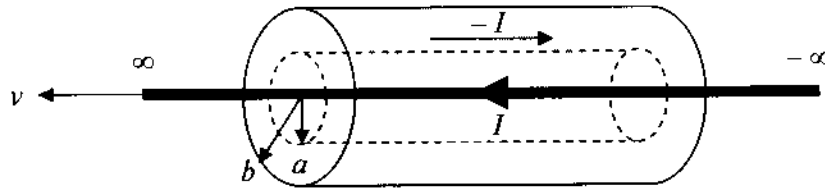
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**FIGURE Q3(b)****FIGURE Q3(c)**

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**FIGURE Q4(a)****FIGURE Q4(b)**

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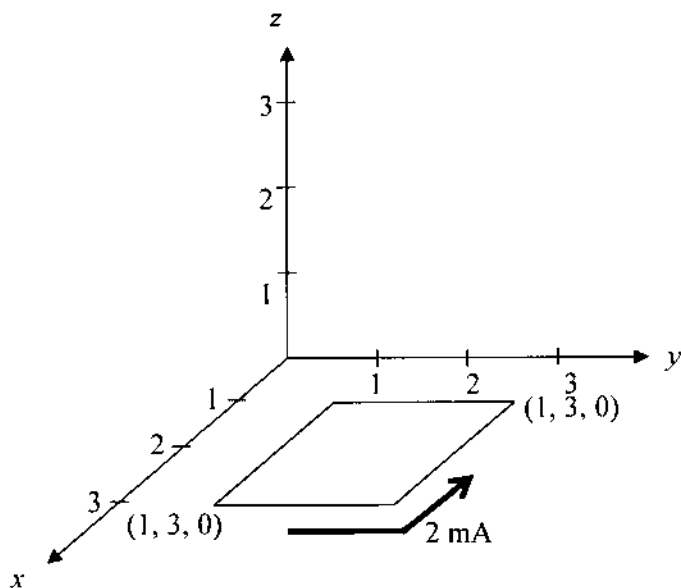


FIGURE Q5(b)

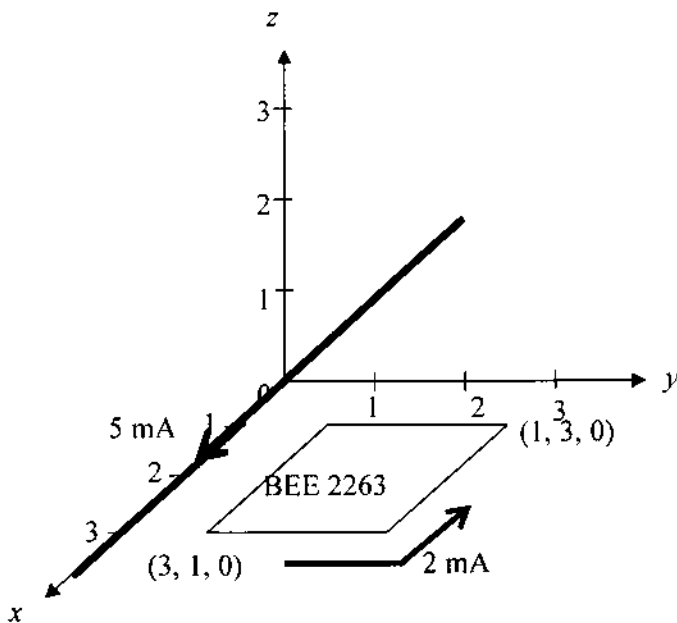
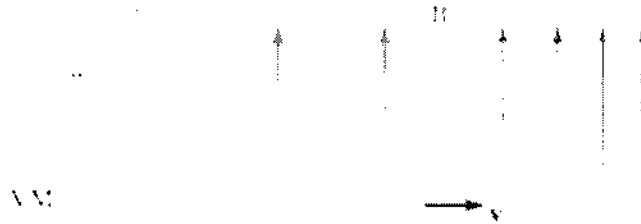
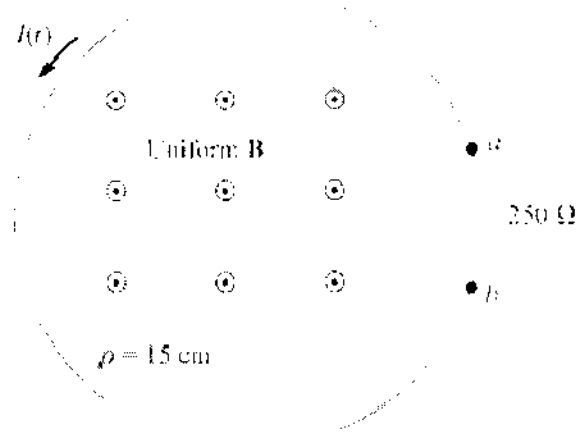


FIGURE Q5(b)(ii)

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**FIGURE Q6(b)****FIGURE Q6(c)**

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FORMULA

Gradient

$$\nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

$$\nabla f = \frac{\partial f}{\partial R} \hat{R} + \frac{1}{R} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{R \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

Divergence

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \left[\frac{\partial(rA_r)}{\partial r} \right] + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial(R^2 A_R)}{\partial R} + \frac{1}{R \sin \theta} \left[\frac{\partial(A_\theta \sin \theta)}{\partial \theta} \right] + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \hat{\phi} + \frac{1}{r} \left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi} \right) \hat{z}$$

$$\nabla \times \vec{A} = \frac{1}{R \sin \theta} \left[\frac{\partial(\sin \theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \hat{R} + \frac{1}{R} \left[\frac{1}{\sin \theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial(RA_\phi)}{\partial R} \right] \hat{\theta} + \frac{1}{R} \left[\frac{\partial(RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta} \right] \hat{\phi}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial f}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \left(\frac{\partial^2 f}{\partial \phi^2} \right)$$

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	Cartesian	Cylindrical	Spherical
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector \vec{A}	$A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$	$A_r \hat{r} + A_\phi \hat{\phi} + A_z \hat{z}$	$A_R \hat{R} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$
Magnitude $ \vec{A} $	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector, \vec{OP}	$x_1 \hat{x} + y_1 \hat{y} + z_1 \hat{z}$ for point $P(x_1, y_1, z_1)$	$r_1 \hat{r} + z_1 \hat{z}$ for point $P(r_1, \phi_1, z_1)$	$R_1 \hat{R}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$ $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{x} = 0$ $\hat{x} \times \hat{y} = \hat{z}$ $\hat{y} \times \hat{z} = \hat{x}$ $\hat{z} \times \hat{x} = \hat{y}$	$\hat{r} \cdot \hat{r} = \hat{\phi} \cdot \hat{\phi} = \hat{z} \cdot \hat{z} = 1$ $\hat{r} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{z} = \hat{z} \cdot \hat{r} = 0$ $\hat{r} \times \hat{\phi} = \hat{z}$ $\hat{\phi} \times \hat{z} = \hat{r}$ $\hat{z} \times \hat{r} = \hat{\phi}$	$\hat{R} \cdot \hat{R} = \hat{\theta} \cdot \hat{\theta} = \hat{\phi} \cdot \hat{\phi} = 1$ $\hat{R} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{R} = 0$ $\hat{R} \times \hat{\theta} = \hat{\phi}$ $\hat{\theta} \times \hat{\phi} = \hat{R}$ $\hat{\phi} \times \hat{R} = \hat{\theta}$
Dot product $\vec{A} \cdot \vec{B}$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A} \times \vec{B}$	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\ell$	$dx \hat{x} + dy \hat{y} + dz \hat{z}$	$dr \hat{r} + r d\phi \hat{\phi} + dz \hat{z}$	$dR \hat{R} + R d\theta \hat{\theta} + R \sin \theta d\phi \hat{\phi}$
Differential surface, \vec{ds}	$\vec{ds}_x = dy dz \hat{x}$ $\vec{ds}_y = dx dz \hat{y}$ $\vec{ds}_z = dx dy \hat{z}$	$\vec{ds}_r = rd\phi dz \hat{r}$ $\vec{ds}_\phi = dr dz \hat{\phi}$ $\vec{ds}_z = r dr d\phi \hat{z}$	$\vec{ds}_R = R^2 \sin \theta d\theta d\phi \hat{R}$ $\vec{ds}_\theta = R \sin \theta dR d\phi \hat{\theta}$ $\vec{ds}_\phi = R dR d\theta \hat{\phi}$
Differential volume, \vec{dv}	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin \theta dR d\theta d\phi$

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Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

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$$Q = \int \rho_v d\ell$$

$$Q = \int \rho_s dS$$

$$Q = \int \rho_v dv$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{a}_{R_{12}}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_R$$

$$\vec{E} = \int \frac{\rho_v d\ell}{4\pi\epsilon_0 R^2} \hat{a}_R$$

$$\vec{E} = \int \frac{\rho_s dS}{4\pi\epsilon_0 R^2} \hat{a}_R$$

$$\vec{E} = \int \frac{\rho_v dv}{4\pi\epsilon_0 R^2} \hat{a}_R$$

$$\vec{D} = \epsilon \vec{E}$$

$$\psi_e = \int \vec{D} \cdot d\vec{S}$$

$$Q_{enc} = \oint \vec{D} \cdot d\vec{S}$$

$$\rho_v = \nabla \cdot \vec{D}$$

$$V_{AB} = - \int_A^B \vec{E} \cdot d\vec{\ell} = \frac{W}{Q}$$

$$V = \frac{Q}{4\pi\epsilon r}$$

$$V = \int \frac{\rho_v d\ell}{4\pi\epsilon r}$$

$$\oint \vec{E} \cdot d\vec{\ell} = 0$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\nabla^2 V = 0$$

$$R = \frac{\ell}{\sigma S}$$

$$I = \int \vec{J} \cdot d\vec{S}$$

$$d\vec{H} = \frac{I d\vec{\ell} \times \vec{R}}{4\pi R^3}$$

$$I d\vec{\ell} = \vec{J}_s dS = \vec{J} dv$$

$$\oint \vec{H} \cdot d\vec{\ell} = I_{enc} = \int \vec{J}_s dS$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\psi_m = \int \vec{B} \cdot d\vec{S}$$

$$\psi_m = \oint \vec{B} \cdot d\vec{S} = 0$$

$$\psi_m = \oint \vec{A} \cdot d\vec{\ell}$$

$$\nabla \cdot \vec{B} = 0$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{A} = \int \frac{\mu_0 I d\vec{\ell}}{4\pi R}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B}) = m \frac{d\vec{u}}{dt}$$

$$d\vec{F} = I d\vec{\ell} \times \vec{B}$$

$$\vec{T} = \vec{r} \times \vec{F} = \vec{m} \times \vec{B}$$

$$\vec{m} = I S \hat{a}_n$$

$$V_{emf} = - \frac{\partial \psi}{\partial t}$$

$$V_{emf} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$V_{emf} = \int (\vec{u} \times \vec{B}) \cdot d\vec{\ell}$$

$$I_d = \int \vec{J}_d \cdot d\vec{S}, J_d = \frac{\partial \vec{D}}{\partial t}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left[\frac{\sigma}{\omega\epsilon} \right]^2} - 1 \right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left[\sqrt{1 + \left[\frac{\sigma}{\omega\epsilon} \right]^2} + 1 \right]}$$

$$\vec{F}_1 = \frac{\mu I_1 I_2}{4\pi} \oint \oint \frac{d\vec{\ell}_1 \times (d\vec{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/4}}$$

$$\tan 2\theta_n = \frac{\sigma}{\omega\epsilon}$$

$$\tan \theta = \frac{\sigma}{\omega\epsilon} = \frac{\vec{J}_s}{\vec{J}_{ds}}$$

$$\delta = \frac{1}{\alpha}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2(x^2 + c^2)^{1/2}}$$

$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$

$$\int \frac{dx}{(x^2 \pm c^2)^{3/2}} = \ln(x + \sqrt{x^2 \pm c^2})$$

$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c} \tan^{-1} \left(\frac{x}{c} \right)$$

$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2} \ln(x^2 + c^2)$$

$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \sqrt{x^2 + c^2}$$