UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	: ELECTROMAGNETIC FIELDS AND WAVES
COURSE CODE	: BEB 20303/BEE 2263/BEX 20903
PROGRAMME	: BED/BEU/BEB/BEH/BEC/BEE
EXAMINATION DATE	: JUNE 2013
DURATION	: 3 HOURS
INSTRUCTION	: ANSWER **FOUR** (4) QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **FIFTEEN (15)** PAGES

Q1 (a) A steady current flows through a finite length wire. When a compass is brought closer to the wire, the compass needle is deflected.

(i) Explain this phenomenon briefly using the Biot-Savart's law and write the complete law mathematically.

(3 marks)

(ii) If the compass is placed far off from the current-carrying wire, the needle returned to its original position, pointing to the true earth's magnetic pole directions. Why is this happen? Explain your answer by referring to the Biot-Savart's law.

(3 marks)

(b) Consider a finite length wire along the z-axis conducting current I in the $+\hat{a}_z$ direction as shown in Figure Q1 (b). Prove that the magnetic field everywhere is $\vec{H} = \frac{I}{4\pi\rho}[cos\alpha_2 - cos\alpha_1]\hat{a}_\phi$.

(6 marks)

(c) A wire loop ABCD forms a closed-circuit system as shown in Figure Q1 (c). Wire BC is designed to be far off from point P, in comparison with wire AD. The parts of wire AB and CD, when extended will meet at point P.

(i) Calculate the magnetic field intensity at point P due to the current on wire AB and CD.

(4 marks)

(ii) What is the best assumption can you make for the contribution of current from wire BC to the magnetic field intensity at point P. Justify your answer.

(3 marks)

(iii) Based on your answer in Q1 (c) (i) and Q1 (c) (ii), find the total magnetic field intensity $\vec{H}_{Total}$ at point P due to current-carrying wire ABCD, if AD = AP = PD and A(0,0,-1), D(0,0,1).

(6 marks)

Q2 (a) Ampere's law is a special case of Biot Savart's law. State what Ampere's circuit law is and what is the important condition in terms of the current distribution one must consider prior to use the law.

(4 marks)

(b) A hollow conducting cylinder has inner radius a and carries current $+I$ along the positive z-direction. Its outer conductor has inner radius b with thickness t, and carries current $-I$. Assume that the current is uniformly distributed in both conductors.

(i) Find $\vec{H}$ everywhere.

(15 marks)

(ii) Plot the magnitude of $\vec{H}$ versus the radial position.

(3 marks)

(iii) If a compass is brought anywhere closer to the conducting cylinder, what would happen to the pole pointing needle of the compass? Explain your answer.

(3 marks)

Q3 (a) *Michael Faraday* was an English physicist working in the early 1800's. Faraday's big discovery happened in 1831 when he found that a change in magnetic field creates an electric current. He did a lot of other work with electricity such as making generators and experimenting with electrochemistry and electrolysis. Recommend an experiment to demonstrate the application of the Faraday's Law in electrical engineering.

(10 marks)

(b) Faraday's Law states that the induced electromotive force in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit. This is a basic law of electromagnetism relating to the operating principles of transformers. Draw the structure of a transformer and explain its operation.

(10 marks)

(c) A perfectly conducting circular loop of radius 20 cm lies in the $x = 0$ plane in a magnetic field density, $\vec{B} = 10\cos(377t)\hat{x}\, mWb/m^2$. Calculate the induced voltage in the loop.

(5 marks)

Q4 (a) A wire with a current I is placed under a clear sheet of plastic, as shown in Figure Q4 (a). Three loops of wire, A, B, and C, are placed on the sheet of plastic at the indicated locations. If the current in the wire is increased, indicate whether the induced electromotive force (EMF) in each of the loops is clockwise, counterclockwise, or zero. Justify your answer for each loop.

(6 marks)

(b) Figure Q4 (b) shows an airplane with wingspan 39.9 m flying northward at 850 km/h through a magnetic field with vertical component of $B = 5 \times 10^{-6}\,T$. Calculate the induced electromotive force (EMF) between the wing tips. Briefly explain why the horizontal component of the Earth's magnetic field does not contribute to the EMF between the wing tips of the airplane.

(7 marks)

(c) A solenoid has an inductance $L = 3.1$ H and carries a current of $I = 15$ A.

(i) If the current goes from 15 to 0 A in a time of 75 ms, analyse the emf induced in the solenoid?

(3 marks)

(ii) Calculate the electrical energy stored in the solenoid?

(2 marks)

(iii) At what rate must the electrical energy be removed from the solenoid when the current is reduced to zero in 75 ms?

(2 marks)

(d) A coil of radius 15 cm and 53 turns is oriented perpendicular to a magnetic field. The magnetic field changes from 0.45 T to zero in 0.12s. Calculate the induced EMF.

(5 marks)

Q5 (a) An engineer decided to shield an Intensive Care Unit (ICU) in a hospital with some lossy material to absorb some electromagnetic fields generated by nearby electrical equipment. Explain the term lossy material in terms of its conductivity, permittivity and permeability.

(6 marks)

(b) The engineer discovered a plane wave propagating through the dielectric, at a particular radian frequency ω, has a magnetic field component

$$5e^{-\alpha x} cos\left(\omega t - \frac{1}{4}x\right) a_y \ ^A/_m$$

(i) Determine **E** if the lossy dielectric of his choice has an intrinsic impedance of $100e^{j\pi/6}$ at that particular radian frequency.

(4 marks)

(ii) Investigate why it is important for the engineer to look into the skin depth. Calculate α and then the minimum depth of this material for it to be effective.

(6 marks)

(iii) Define and calculate the lost tangent of the material.

(4 marks)

(iv) Based on the calculated skin depth and loss tangent in Q5 b (ii) and Q5 b (iii) respectively, analyse if the material is suitable for the engineer's application. Determine other parameters that the engineer has to take into account in selecting the right absorber for his application.

(5 marks)

\- **END OF QUESTION** -

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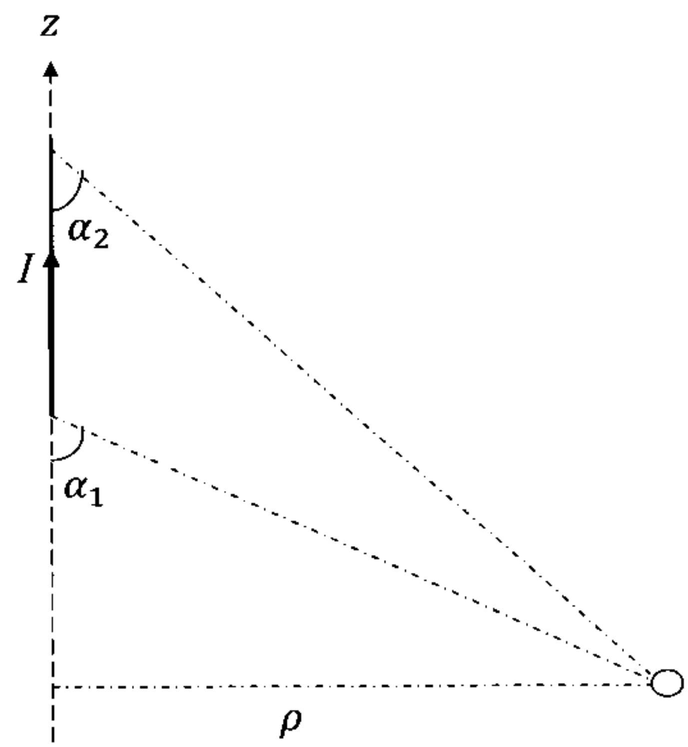

FIGURE Q1 (b)

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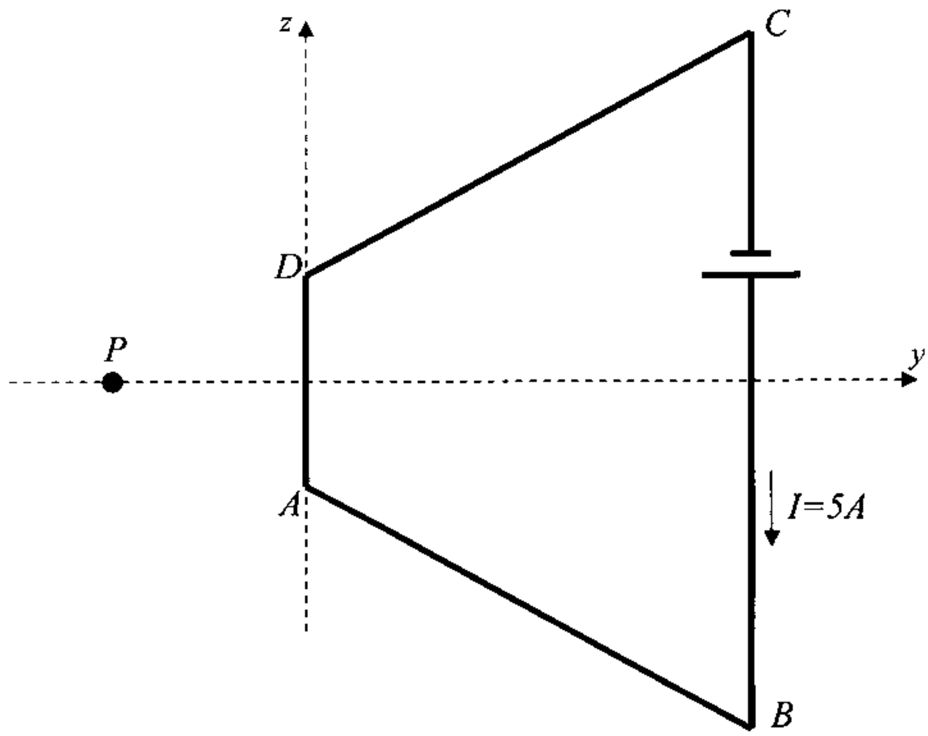

FIGURE Q1 (c)

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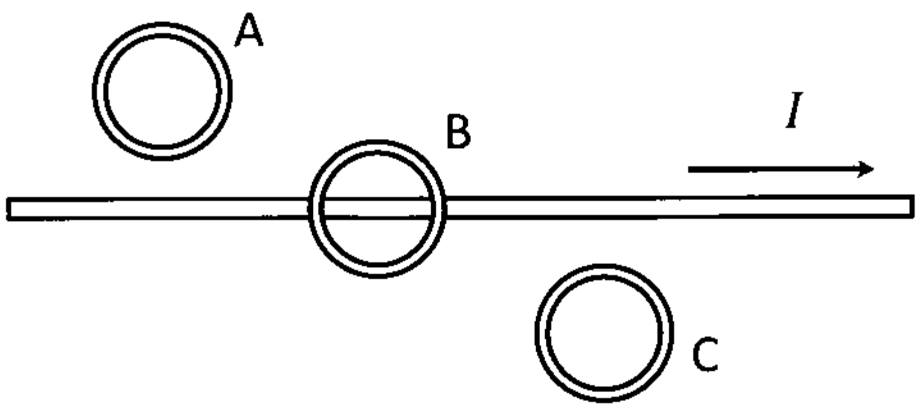

FIGURE Q4 (a)

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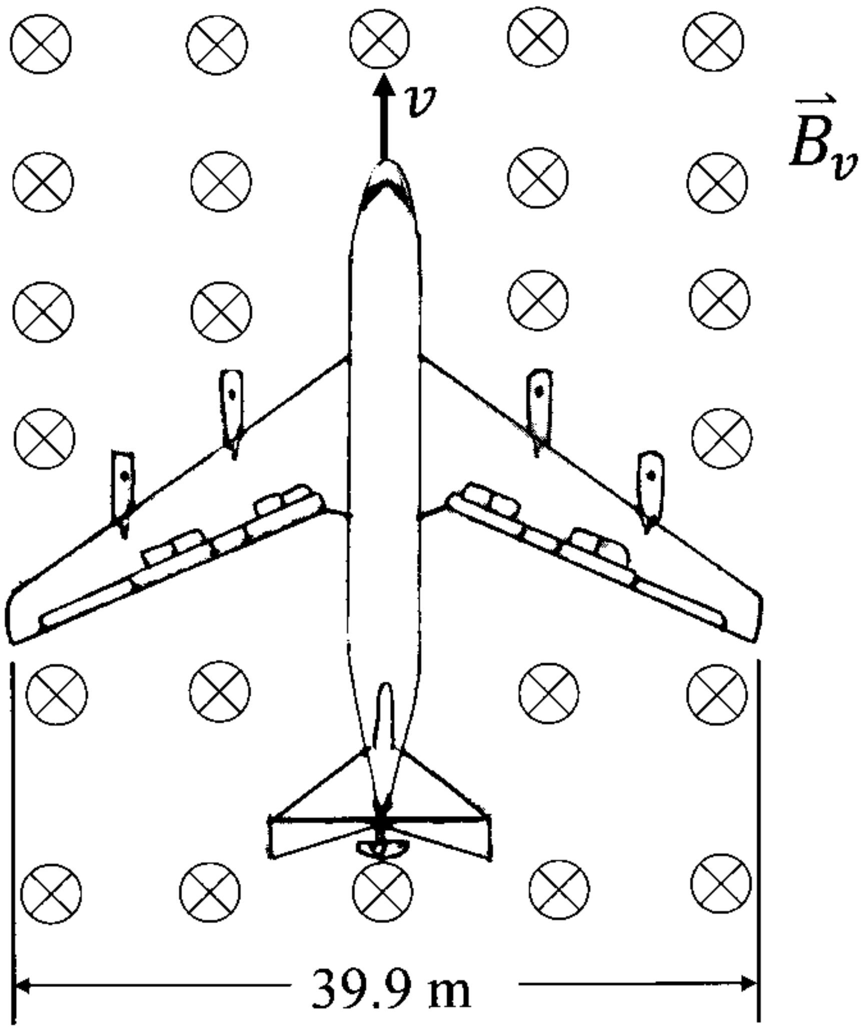

FIGURE Q4 (b)

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Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

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	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\phi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_r^{\ 2} + A_\phi^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_R^{\ 2} + A_\theta^{\ 2} + A_\phi^{\ 2}}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\bullet\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\bullet\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\phi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\phi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\phi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\ \ dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

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$$Q = \int \rho_\ell d\ell,$$
$$Q = \int \rho_s dS,$$
$$Q = \int \rho_v dv$$
$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$
$$\overline{E} = \frac{\overline{F}}{Q},$$
$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{D} = \varepsilon\overline{E}$$
$$\psi_e = \int \overline{D} \bullet d\overline{S}$$
$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$
$$\rho_v = \nabla \bullet \overline{D}$$
$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$
$$V = \frac{Q}{4\pi\varepsilon r}$$
$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint \overline{E} \bullet d\overline{\ell} = 0$$
$$\nabla \times \overline{E} = 0$$
$$\overline{E} = -\nabla V$$
$$\nabla^2 V = 0$$
$$R = \frac{\ell}{\sigma S}$$
$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{I d\overline{\ell} \times \overline{R}}{4\pi R^3}$$
$$I d\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$$
$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$
$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$
$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$
$$\nabla \bullet \overline{B} = 0$$
$$\overline{B} = \mu\overline{H}$$
$$\overline{B} = \nabla \times \overline{A}$$
$$\overline{A} = \int \frac{\mu_0 I d\overline{\ell}}{4\pi R}$$
$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$
$$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\frac{d\overline{u}}{dt}$$
$$d\overline{F} = I d\overline{\ell} \times \overline{B}$$
$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$
$$\overline{m} = IS\hat{a}_n$$
$$V_{emf} = -\frac{\partial\psi}{\partial t}$$
$$V_{emf} = -\int \frac{\partial\overline{B}}{\partial t} \bullet d\overline{S}$$
$$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$
$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial\overline{D}}{\partial t}$$
$$\gamma = \alpha + j\beta$$
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$
$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2} \frac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$
$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$tan\, 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$
$$tan\, \theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta = \frac{1}{\alpha}$$
$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$
$$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2 (x^2 + c^2)^{1/2}}$$
$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$
$$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$$
$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c} tan^{-1}\left(\frac{x}{c}\right)$$
$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2} ln(x^2 + c^2)$$
$$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$$